What you should learn from Recitation 7: a.Finding Particular Solutions of Inhomogenous ODE

Fei Qi

Rutgers University

fq15@math.rutgers.edu

March 13, 2014

Disclaimer

- The slides are intended to serve as records for a recitation for math 244 course. It should never serve as any replacement for formal lectures or as any reviewing material. The author is not responsible for consequences brought by inappropriate use.
- There may be errors. Use them at your own discretion. Anyone who notify me with an error will get some award in grade points.

Special Announcement

- Please watch MIT Lecture 13 before reading my slides.

Special Announcement

- Please watch MIT Lecture 13 before reading my slides.
- I will only use the Exponentila-Shift Rule mentioned in the MIT lecture, as it applies to all our cases.

Special Announcement

- Please watch MIT Lecture 13 before reading my slides.
- I will only use the Exponentila-Shift Rule mentioned in the MIT lecture, as it applies to all our cases. I won't use anything else.

Special Announcement

- Please watch MIT Lecture 13 before reading my slides.
- I will only use the Exponentila-Shift Rule mentioned in the MIT lecture, as it applies to all our cases. I won't use anything else. In particular, the method of complexification will not be used.

Special Announcement

- Please watch MIT Lecture 13 before reading my slides.
- I will only use the Exponentila-Shift Rule mentioned in the MIT lecture, as it applies to all our cases. I won't use anything else. In particular, the method of complexification will not be used. You don't have to know that as well.

Special Announcement

- Please watch MIT Lecture 13 before reading my slides.
- I will only use the Exponentila-Shift Rule mentioned in the MIT lecture, as it applies to all our cases. I won't use anything else. In particular, the method of complexification will not be used. You don't have to know that as well.
- Although the instructor skipped variation of parameter,

Special Announcement

- Please watch MIT Lecture 13 before reading my slides.
- I will only use the Exponentila-Shift Rule mentioned in the MIT lecture, as it applies to all our cases. I won't use anything else. In particular, the method of complexification will not be used. You don't have to know that as well.
- Although the instructor skipped variation of parameter, I will talk about it in additional slides,

Special Announcement

- Please watch MIT Lecture 13 before reading my slides.
- I will only use the Exponentila-Shift Rule mentioned in the MIT lecture, as it applies to all our cases. I won't use anything else. In particular, the method of complexification will not be used. You don't have to know that as well.
- Although the instructor skipped variation of parameter, I will talk about it in additional slides, as the method is very general

Special Announcement

- Please watch MIT Lecture 13 before reading my slides.
- I will only use the Exponentila-Shift Rule mentioned in the MIT lecture, as it applies to all our cases. I won't use anything else. In particular, the method of complexification will not be used. You don't have to know that as well.
- Although the instructor skipped variation of parameter, I will talk about it in additional slides, as the method is very general and actually reduction of order can be treated as an application.

Special Announcement

- Please watch MIT Lecture 13 before reading my slides.
- I will only use the Exponentila-Shift Rule mentioned in the MIT lecture, as it applies to all our cases. I won't use anything else. In particular, the method of complexification will not be used. You don't have to know that as well.
- Although the instructor skipped variation of parameter, I will talk about it in additional slides, as the method is very general and actually reduction of order can be treated as an application. If you are not interested, just don't look at it.

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

the general solution looks like

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + P(t),$$

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

the general solution looks like

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + P(t),$$

where

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

the general solution looks like

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + P(t),$$

where

- $c_1 y_1(t) + c_2 y_2(t)$ are called complementary solutions

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

the general solution looks like

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + P(t),$$

where

- $c_1 y_1(t) + c_2 y_2(t)$ are called complementary solutions and it is precisely the general solution to $ay'' + by' + cy = 0$.

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

the general solution looks like

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + P(t),$$

where

- $c_1 y_1(t) + c_2 y_2(t)$ are called complementary solutions and it is precisely the general solution to $ay'' + by' + cy = 0$.
- $P(t)$ is a particular solution.

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

the general solution looks like

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + P(t),$$

where

- $c_1 y_1(t) + c_2 y_2(t)$ are called complementary solutions and it is precisely the general solution to $ay'' + by' + cy = 0$.
- $P(t)$ is a particular solution. In other words, $aP''(t) + bP'(t) + cP(t) = g(t)$.

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

the general solution looks like

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + P(t),$$

where

- $c_1 y_1(t) + c_2 y_2(t)$ are called complementary solutions and it is precisely the general solution to $ay'' + by' + cy = 0$.
- $P(t)$ is a particular solution. In other words, $aP''(t) + bP'(t) + cP(t) = g(t)$.

If the $g(t)$ is special enough,

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

the general solution looks like

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + P(t),$$

where

- $c_1 y_1(t) + c_2 y_2(t)$ are called complementary solutions and it is precisely the general solution to $ay'' + by' + cy = 0$.
- $P(t)$ is a particular solution. In other words, $aP''(t) + bP'(t) + cP(t) = g(t)$.

If the $g(t)$ is special enough, then you can proceed by guessing the solution.

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

the general solution looks like

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + P(t),$$

where

- $c_1 y_1(t) + c_2 y_2(t)$ are called complementary solutions and it is precisely the general solution to $ay'' + by' + cy = 0$.
- $P(t)$ is a particular solution. In other words, $aP''(t) + bP'(t) + cP(t) = g(t)$.

If the $g(t)$ is special enough, then you can proceed by guessing the solution. Otherwise, you have to use variation of parameters,

Structure of the solution

For the second order linear inhomogeneous ODE

$$ay'' + by' + cy = g(t),$$

the general solution looks like

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + P(t),$$

where

- $c_1 y_1(t) + c_2 y_2(t)$ are called complementary solutions and it is precisely the general solution to $ay'' + by' + cy = 0$.
- $P(t)$ is a particular solution. In other words, $aP''(t) + bP'(t) + cP(t) = g(t)$.

If the $g(t)$ is special enough, then you can proceed by guessing the solution. Otherwise, you have to use variation of parameters, which works for all cases but usually is less convenient.

How to Guess a particular solution

The template for your first try is summarized as below:

How to Guess a particular solution

The template for your first try is summarized as below:

- If $g(t)$ looks like

$$g(t) = e^{\alpha t}(a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0),$$

How to Guess a particular solution

The template for your first try is summarized as below:

- If $g(t)$ looks like

$$g(t) = e^{\alpha t}(a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0),$$

then try

$$P(t) = e^{\alpha t}(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0),$$

How to Guess a particular solution

The template for your first try is summarized as below:

- If $g(t)$ looks like

$$g(t) = e^{\alpha t}(a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0),$$

  then try

$$P(t) = e^{\alpha t}(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0),$$

  where $A_n, \cdots, A_0$ are coefficients to be determined.

How to Guess a particular solution

The template for your first try is summarized as below:

- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0),$$
then try
$$P(t) = e^{\alpha t}(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0),$$
where $A_n, \cdots, A_0$ are coefficients to be determined.
- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a \cos \beta t + b \sin \beta t),$$

How to Guess a particular solution

The template for your first try is summarized as below:

- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0),$$
then try
$$P(t) = e^{\alpha t}(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0),$$
where $A_n, \cdots, A_0$ are coefficients to be determined.
- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a \cos \beta t + b \sin \beta t),$$
then try
$$P(t) = e^{\alpha t}(A \cos \beta t + B \sin \beta t)$$

How to Guess a particular solution

The template for your first try is summarized as below:

- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0),$$
then try
$$P(t) = e^{\alpha t}(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0),$$
where $A_n, \cdots, A_0$ are coefficients to be determined.
- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a \cos \beta t + b \sin \beta t),$$
then try
$$P(t) = e^{\alpha t}(A \cos \beta t + B \sin \beta t)$$
where A, B are coefficients to be determined.

How to Guess a particular solution

The template for your first try is summarized as below:

- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a_n t^n + a_{n-1}t^{n-1} + \cdots + a_0),$$
then try
$$P(t) = e^{\alpha t}(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_0),$$
where $A_n, \cdots, A_0$ are coefficients to be determined.
- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a \cos \beta t + b \sin \beta t),$$
then try
$$P(t) = e^{\alpha t}(A \cos \beta t + B \sin \beta t)$$
where A, B are coefficients to be determined.

If your first try fails,

How to Guess a particular solution

The template for your first try is summarized as below:

- If $g(t)$ looks like

  $$g(t) = e^{\alpha t}(a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0),$$

  then try

  $$P(t) = e^{\alpha t}(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0),$$

  where $A_n, \cdots, A_0$ are coefficients to be determined.
- If $g(t)$ looks like

  $$g(t) = e^{\alpha t}(a \cos \beta t + b \sin \beta t),$$

  then try

  $$P(t) = e^{\alpha t}(A \cos \beta t + B \sin \beta t)$$

  where A, B are coefficients to be determined.

If your first try fails, multiply your template with t

How to Guess a particular solution

The template for your first try is summarized as below:

- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0),$$
then try
$$P(t) = e^{\alpha t}(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0),$$
where $A_n, \cdots, A_0$ are coefficients to be determined.
- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a \cos \beta t + b \sin \beta t),$$
then try
$$P(t) = e^{\alpha t}(A \cos \beta t + B \sin \beta t)$$
where A, B are coefficients to be determined.

If your first try fails, multiply your template with t and try again.

How to Guess a particular solution

The template for your first try is summarized as below:

- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a_n t^n + a_{n-1}t^{n-1} + \cdots + a_0),$$
then try
$$P(t) = e^{\alpha t}(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_0),$$
where $A_n, \cdots, A_0$ are coefficients to be determined.
- If $g(t)$ looks like
$$g(t) = e^{\alpha t}(a\cos\beta t + b\sin\beta t),$$
then try
$$P(t) = e^{\alpha t}(A\cos\beta t + B\sin\beta t)$$
where A, B are coefficients to be determined.

If your first try fails, multiply your template with t and try again. If the second try fails, multiply your template with another t and try again. It is guaranteed that after finite attempts you will suceed.

Examples and remarks

- $g(t) = 3$.

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$.

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$,

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t + 2)$, i.e., $\alpha = 1$,

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$, i.e., $\alpha = 1, n = 1$,

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$.

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$,

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t + 2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3$,

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t + 2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3, n = 2$,

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t + 2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3, n = 2, a_2 = 1, a_1 = a_0 = 0$.

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t + 2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3, n = 2, a_2 = 1, a_1 = a_0 = 0$. Try the template $P(t) = e^t(A_2 t^2 + A_1 t + A_0)$.

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3, n = 2, a_2 = 1, a_1 = a_0 = 0$. Try the template $P(t) = e^t(A_2 t^2 + A_1 t + A_0)$.
  CAUTIOUS:

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3, n = 2, a_2 = 1, a_1 = a_0 = 0$. Try the template $P(t) = e^t(A_2 t^2 + A_1 t + A_0)$.
  CAUTIOUS: Even you have only 1 term,

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3, n = 2, a_2 = 1, a_1 = a_0 = 0$. Try the template $P(t) = e^t(A_2 t^2 + A_1 t + A_0)$.
  CAUTIOUS: Even you have only 1 term, since your polynomial is of degree 2,

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3, n = 2, a_2 = 1, a_1 = a_0 = 0$. Try the template $P(t) = e^t(A_2 t^2 + A_1 t + A_0)$.
  CAUTIOUS: Even you have only 1 term, since your polynomial is of degree 2, THERE SHOULD BE 3 UNDETERMINED COEFFICIENTS.

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3, n = 2, a_2 = 1, a_1 = a_0 = 0$. Try the template $P(t) = e^t(A_2 t^2 + A_1 t + A_0)$.
  CAUTIOUS: Even you have only 1 term, since your polynomial is of degree 2, THERE SHOULD BE 3 UNDETERMINED COEFFICIENTS. In general if your polynomial is of degree n

Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t+2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3, n = 2, a_2 = 1, a_1 = a_0 = 0$. Try the template $P(t) = e^t(A_2 t^2 + A_1 t + A_0)$.
  CAUTIOUS: Even you have only 1 term, since your polynomial is of degree 2, THERE SHOULD BE 3 UNDETERMINED COEFFICIENTS. In general if your polynomial is of degree n, there should be $n+1$ of undetermined coefficients,

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$.
- $g(t) = e^t(t + 2)$, i.e., $\alpha = 1, n = 1, a_1 = 1, a_0 = 2$. Try the template $P(t) = e^t(A_1 t + A_0)$.
- $g(t) = t^2 e^{3t}$, i.e., $\alpha = 3, n = 2, a_2 = 1, a_1 = a_0 = 0$. Try the template $P(t) = e^t(A_2 t^2 + A_1 t + A_0)$.
  CAUTIOUS: Even you have only 1 term, since your polynomial is of degree 2, THERE SHOULD BE 3 UNDETERMINED COEFFICIENTS. In general if your polynomial is of degree n, there should be $n + 1$ of undetermined coefficients, REGARDLESS how many the lower terms there are.

Examples and remarks

- Another example of this type:

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$,

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial,

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial, the template should be $P(t) = e^t(A_4 t^4 + A_3 t^3 + A_2 t^2 + A_1 t^1 + A_0)$.

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial, the template should be $P(t) = e^t(A_4 t^4 + A_3 t^3 + A_2 t^2 + A_1 t^1 + A_0)$. The lower terms DON'T MATTER here.

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial, the template should be $P(t) = e^t(A_4t^4 + A_3t^3 + A_2t^2 + A_1t^1 + A_0)$. The lower terms DON'T MATTER here.
- $g(t) = e^{6t}(2\cos t + 3\sin t)$.

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial, the template should be $P(t) = e^t(A_4 t^4 + A_3 t^3 + A_2 t^2 + A_1 t^1 + A_0)$. The lower terms DON'T MATTER here.
- $g(t) = e^{6t}(2\cos t + 3\sin t)$. Try the template $P(t) = e^{6t}(A\cos t + B\sin t)$.

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial, the template should be $P(t) = e^t(A_4 t^4 + A_3 t^3 + A_2 t^2 + A_1 t^1 + A_0)$. The lower terms DON'T MATTER here.
- $g(t) = e^{6t}(2\cos t + 3\sin t)$. Try the template $P(t) = e^{6t}(A\cos t + B\sin t)$.
- $g(t) = e^{3t}\cos t$.

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial, the template should be $P(t) = e^t(A_4t^4 + A_3t^3 + A_2t^2 + A_1t^1 + A_0)$. The lower terms DON'T MATTER here.
- $g(t) = e^{6t}(2\cos t + 3\sin t)$. Try the template $P(t) = e^{6t}(A\cos t + B\sin t)$.
- $g(t) = e^{3t}\cos t$. Although you have only one term,

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial, the template should be $P(t) = e^t(A_4t^4 + A_3t^3 + A_2t^2 + A_1t^1 + A_0)$. The lower terms DON'T MATTER here.
- $g(t) = e^{6t}(2\cos t + 3\sin t)$. Try the template $P(t) = e^{6t}(A\cos t + B\sin t)$.
- $g(t) = e^{3t}\cos t$. Although you have only one term, you still have to try $P(t) = e^{3t}(A\cos t + B\sin t)$.

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial, the template should be $P(t) = e^t(A_4t^4 + A_3t^3 + A_2t^2 + A_1t^1 + A_0)$. The lower terms DON'T MATTER here.
- $g(t) = e^{6t}(2\cos t + 3\sin t)$. Try the template $P(t) = e^{6t}(A\cos t + B\sin t)$.
- $g(t) = e^{3t}\cos t$. Although you have only one term, you still have to try $P(t) = e^{3t}(A\cos t + B\sin t)$.
- $g(t) = e^t t^2 \cos t$.

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial, the template should be $P(t) = e^t(A_4t^4 + A_3t^3 + A_2t^2 + A_1t^1 + A_0)$. The lower terms DON'T MATTER here.
- $g(t) = e^{6t}(2\cos t + 3\sin t)$. Try the template $P(t) = e^{6t}(A\cos t + B\sin t)$.
- $g(t) = e^{3t}\cos t$. Although you have only one term, you still have to try $P(t) = e^{3t}(A\cos t + B\sin t)$.
- $g(t) = e^t t^2 \cos t$. This case is in fact too complicated to use this method.

Examples and remarks

- Another example of this type: $g(t) = (t^4 + t)e^{3t}$, since you have a degree 4 polynomial, the template should be $P(t) = e^t(A_4 t^4 + A_3 t^3 + A_2 t^2 + A_1 t^1 + A_0)$. The lower terms DON'T MATTER here.
- $g(t) = e^{6t}(2\cos t + 3\sin t)$. Try the template $P(t) = e^{6t}(A\cos t + B\sin t)$.
- $g(t) = e^{3t}\cos t$. Although you have only one term, you still have to try $P(t) = e^{3t}(A\cos t + B\sin t)$.
- $g(t) = e^t t^2 \cos t$. This case is in fact too complicated to use this method. Instead, use variation of parameters.

Facts that will be used in solving problems

- General fact:

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$,

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$,

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.
- Exercise: Prove this fact. (very easy)

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.
- Exercise: Prove this fact. (very easy)
- So for example,

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.
- Exercise: Prove this fact. (very easy)
- So for example, if the ODE looks like

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.
- Exercise: Prove this fact. (very easy)
- So for example, if the ODE looks like

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

then you should find particular solutions

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.
- Exercise: Prove this fact. (very easy)
- So for example, if the ODE looks like

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

then you should find particular solutions
$P_1(t)$ for $y'' + 2y' - 3y = e^t(t^2 + 4)$,

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.
- Exercise: Prove this fact. (very easy)
- So for example, if the ODE looks like

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

then you should find particular solutions
$P_1(t)$ for $y'' + 2y' - 3y = e^t(t^2 + 4)$,
$P_2(t)$ for $y'' + 2y' - 3y = e^{3t}\cos 3t$,

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.
- Exercise: Prove this fact. (very easy)
- So for example, if the ODE looks like

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

then you should find particular solutions
$P_1(t)$ for $y'' + 2y' - 3y = e^t(t^2 + 4)$,
$P_2(t)$ for $y'' + 2y' - 3y = e^{3t}\cos 3t$,
$P_3(t)$ for $y'' + 2y' - 3y = \cos 4t$,

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.
- Exercise: Prove this fact. (very easy)
- So for example, if the ODE looks like

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

then you should find particular solutions
$P_1(t)$ for $y'' + 2y' - 3y = e^t(t^2 + 4)$,
$P_2(t)$ for $y'' + 2y' - 3y = e^{3t}\cos 3t$,
$P_3(t)$ for $y'' + 2y' - 3y = \cos 4t$, and
$P_4(t)$ for $y'' + 2y' - 3y = t^2$.

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.
- Exercise: Prove this fact. (very easy)
- So for example, if the ODE looks like

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

then you should find particular solutions
$P_1(t)$ for $y'' + 2y' - 3y = e^t(t^2 + 4)$,
$P_2(t)$ for $y'' + 2y' - 3y = e^{3t}\cos 3t$,
$P_3(t)$ for $y'' + 2y' - 3y = \cos 4t$, and
$P_4(t)$ for $y'' + 2y' - 3y = t^2$.
Then $P(t) = P_1(t) + P_2(t) + P_3(t) + P_4(t)$

Facts that will be used in solving problems

- General fact: If $P_1(t)$ is a particular solution for $ay'' + by' + cy = g_1(t)$, and $P_2(t)$ is a particular solution for $ay'' + by' + cy = g_2(t)$, then $P_1(t) + P_2(t)$ is a particular solution for $ay'' + by' + cy = g_1(t) + g_2(t)$.
- Exercise: Prove this fact. (very easy)
- So for example, if the ODE looks like

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

then you should find particular solutions
$P_1(t)$ for $y'' + 2y' - 3y = e^t(t^2 + 4)$,
$P_2(t)$ for $y'' + 2y' - 3y = e^{3t}\cos 3t$,
$P_3(t)$ for $y'' + 2y' - 3y = \cos 4t$, and
$P_4(t)$ for $y'' + 2y' - 3y = t^2$.
Then $P(t) = P_1(t) + P_2(t) + P_3(t) + P_4(t)$ is a particular solution for this ODE.

Facts that will be used in solving problems

- A tedious yet complete and comprehensive summary can be found on Page 181 of the book.

Facts that will be used in solving problems

- A tedious yet complete and comprehensive summary can be found on Page 181 of the book.
- I will show the full solution for this ridiculously fabricated ODE

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2.$$

Facts that will be used in solving problems

- A tedious yet complete and comprehensive summary can be found on Page 181 of the book.
- I will show the full solution for this ridiculously fabricated ODE

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2.$$

However in industry, $g(t)$ may have 40 to 50 summands

Facts that will be used in solving problems

- A tedious yet complete and comprehensive summary can be found on Page 181 of the book.
- I will show the full solution for this ridiculously fabricated ODE

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2.$$

However in industry, $g(t)$ may have 40 to 50 summands and that's why you should learn how to use mathematica / maple.

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras.

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture).

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

Theorem

Let $f(x)$ be a polynomial,

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

Theorem

Let $f(x)$ be a polynomial, Denote $D = \frac{d}{dt}$, the derivative operator.

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

Theorem

Let $f(x)$ be a polynomial, Denote $D = \frac{d}{dt}$, the derivative operator. Then for any function $u(t)$

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

Theorem

Let $f(x)$ be a polynomial, Denote $D = \frac{d}{dt}$, the derivative operator. Then for any function $u(t)$ and for any number a,

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

Theorem

Let $f(x)$ be a polynomial, Denote $D = \frac{d}{dt}$, the derivative operator. Then for any function $u(t)$ and for any number a,

$$f(D)(e^{at}u(t)) = e^{at}f(D+a)u(t).$$

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

Theorem

Let $f(x)$ be a polynomial, Denote $D = \frac{d}{dt}$, the derivative operator. Then for any function $u(t)$ and for any number a,

$$f(D)(e^{at}u(t)) = e^{at}f(D+a)u(t).$$

In words, to move e^{at}

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

Theorem

Let $f(x)$ be a polynomial, Denote $D = \frac{d}{dt}$, the derivative operator. Then for any function $u(t)$ and for any number a,

$$f(D)(e^{at}u(t)) = e^{at}f(D+a)u(t).$$

In words, to move e^{at} out of $f(D)(e^{at}u(t))$,

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

Theorem

Let $f(x)$ be a polynomial, Denote $D = \frac{d}{dt}$, the derivative operator. Then for any function $u(t)$ and for any number a,

$$f(D)(e^{at}u(t)) = e^{at}f(D+a)u(t).$$

In words, to move e^{at} out of $f(D)(e^{at}u(t))$, you should pay the price

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

Theorem

Let $f(x)$ be a polynomial, Denote $D = \frac{d}{dt}$, the derivative operator. Then for any function $u(t)$ and for any number a,

$$f(D)(e^{at}u(t)) = e^{at}f(D+a)u(t).$$

In words, to move e^{at} out of $f(D)(e^{at}u(t))$, you should pay the price of modifying $f(D)$ into $f(D+a)$,

Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras. And Dr. Mattuck's Exponential-Shift Rule actually helps simplify the computation (although you don't see any directly-related examples in his lecture). The rule is expressed as follows:

Theorem

Let $f(x)$ be a polynomial, Denote $D = \frac{d}{dt}$, the derivative operator. Then for any function $u(t)$ and for any number a,

$$f(D)(e^{at}u(t)) = e^{at}f(D+a)u(t).$$

In words, to move e^{at} out of $f(D)(e^{at}u(t))$, you should pay the price of modifying $f(D)$ into $f(D+a)$, getting $e^{at}(f(D+a)u(t))$.

Exponential-Shift Rule

Remarks:

Exponential-Shift Rule

Remarks:

- Please check 27:00 of the MIT Lecture 13 for a proof.

Exponential-Shift Rule

Remarks:

- Please check 27:00 of the MIT Lecture 13 for a proof. (Note that he used a different symbol p)

Exponential-Shift Rule

Remarks:

- Please check 27:00 of the MIT Lecture 13 for a proof. (Note that he used a different symbol p)
- Please check 0:00 of the MIT Lecture 14 for the interpretation of $f(D)$.

Exponential-Shift Rule

Remarks:

- Please check 27:00 of the MIT Lecture 13 for a proof. (Note that he used a different symbol p)
- Please check 0:00 of the MIT Lecture 14 for the interpretation of $f(D)$.
- In our scenario where the ODE is $ay'' + by' + cy = g(t)$,

Exponential-Shift Rule

Remarks:

- Please check 27:00 of the MIT Lecture 13 for a proof. (Note that he used a different symbol p)
- Please check 0:00 of the MIT Lecture 14 for the interpretation of $f(D)$.
- In our scenario where the ODE is $ay'' + by' + cy = g(t)$, basically we will always take $f(x) = ax^2 + bx + c$.

Exponential-Shift Rule

Remarks:

- Please check 27:00 of the MIT Lecture 13 for a proof. (Note that he used a different symbol p)
- Please check 0:00 of the MIT Lecture 14 for the interpretation of $f(D)$.
- In our scenario where the ODE is $ay'' + by' + cy = g(t)$, basically we will always take $f(x) = ax^2 + bx + c$.
- The most cited example of $f(D)$ acting on a function

Exponential-Shift Rule

Remarks:

- Please check 27:00 of the MIT Lecture 13 for a proof. (Note that he used a different symbol p)
- Please check 0:00 of the MIT Lecture 14 for the interpretation of $f(D)$.
- In our scenario where the ODE is $ay'' + by' + cy = g(t)$, basically we will always take $f(x) = ax^2 + bx + c$.
- The most cited example of $f(D)$ acting on a function is

$$f(D)y$$

Exponential-Shift Rule

Remarks:

- Please check 27:00 of the MIT Lecture 13 for a proof. (Note that he used a different symbol p)
- Please check 0:00 of the MIT Lecture 14 for the interpretation of $f(D)$.
- In our scenario where the ODE is $ay'' + by' + cy = g(t)$, basically we will always take $f(x) = ax^2 + bx + c$.
- The most cited example of $f(D)$ acting on a function is

$$f(D)y = \left(a\frac{d^2}{dt^2} + b\frac{d}{dt} + c\right) y$$

Exponential-Shift Rule

Remarks:

- Please check 27:00 of the MIT Lecture 13 for a proof. (Note that he used a different symbol p)
- Please check 0:00 of the MIT Lecture 14 for the interpretation of $f(D)$.
- In our scenario where the ODE is $ay'' + by' + cy = g(t)$, basically we will always take $f(x) = ax^2 + bx + c$.
- The most cited example of $f(D)$ acting on a function is

$$f(D)y = \left(a\frac{d^2}{dt^2} + b\frac{d}{dt} + c\right) y = ay'' + by' + cy.$$

Exponential-Shift Rule

Remarks:

- Please check 27:00 of the MIT Lecture 13 for a proof. (Note that he used a different symbol p)
- Please check 0:00 of the MIT Lecture 14 for the interpretation of $f(D)$.
- In our scenario where the ODE is $ay'' + by' + cy = g(t)$, basically we will always take $f(x) = ax^2 + bx + c$.
- The most cited example of $f(D)$ acting on a function is

$$f(D)y = \left(a\frac{d^2}{dt^2} + b\frac{d}{dt} + c\right) y = ay'' + by' + cy.$$

- You will see how this technique is used in the following example problems.

The ridiculously fabricated example problem

Find the general solution of

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2.$$

The ridiculously fabricated example problem

Find the general solution of

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2.$$

- The complementary solution for this ODE is

$$C_1 e^t + C_2 e^{-3t}$$

The ridiculously fabricated example problem

Find the general solution of

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2.$$

- The complementary solution for this ODE is

$$C_1 e^t + C_2 e^{-3t}$$

- To get a particular solution, let's get the particular solution for the 4 summands on the right hand side separately.

The ridiculously fabricated example problem

Find the general solution of

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2.$$

- The complementary solution for this ODE is

$$C_1 e^t + C_2 e^{-3t}$$

- To get a particular solution, let's get the particular solution for the 4 summands on the right hand side separately. Let's first deal with the ODE

$$y'' + 2y' - 3y = e^t(t^2 + 4).$$

The ridiculously fabricated example problem

- The template you should try

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys).

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class,

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows
  Let $f(x) = x^2 + 2x - 3$

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows
  Let $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$.

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows
  Let $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$. Then

$$P'' + 2P' - 3P$$

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows
  Let $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$. Then

$$P'' + 2P' - 3P = (D^2 + 2D - 3)P$$

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows
  Let $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$. Then

$$P'' + 2P' - 3P \quad = \quad (D^2 + 2D - 3)P = f(D)P$$

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows
  Let $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$. Then

$$\begin{aligned} P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\ &= f(D)(e^t(At^2 + Bt + C)) \end{aligned}$$

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows
  Let $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$. Then

$$\begin{aligned} P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\ &= f(D)(e^t(At^2 + Bt + C)) \\ &\quad \text{(Use the exponential-shift rule)} \end{aligned}$$

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows
  Let $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$. Then

$$
\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)(e^t(At^2 + Bt + C)) \\
&\quad \text{(Use the exponential-shift rule)} \\
&= e^t f(D+1)(At^2 + Bt + C)
\end{aligned}
$$

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows
  Let $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$. Then

$$\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)(e^t(At^2 + Bt + C)) \\
&\quad \text{(Use the exponential-shift rule)} \\
&= e^t f(D+1)(At^2 + Bt + C) \\
&= e^t (D+1-1)(D+1+3)(At^2 + Bt + C)
\end{aligned}$$

The ridiculously fabricated example problem

- The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I'll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows
  Let $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$. Then

$$\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)(e^t(At^2 + Bt + C)) \\
&\quad \text{(Use the exponential-shift rule)} \\
&= e^t f(D+1)(At^2 + Bt + C) \\
&= e^t(D+1-1)(D+1+3)(At^2 + Bt + C) \\
&= e^t(D^2 + 4D)(At^2 + Bt + C).
\end{aligned}$$

The ridiculously fabricated example problem

- Notice that

The ridiculously fabricated example problem

- Notice that

$$D(At^2 + Bt + C) = (2At + B)$$

The ridiculously fabricated example problem

- Notice that

$$D(At^2 + Bt + C) = (2At + B)$$
$$D^2(At^2 + Bt + C) = 2A.$$

The ridiculously fabricated example problem

- Notice that

$$D(At^2 + Bt + C) = (2At + B)$$
$$D^2(At^2 + Bt + C) = 2A.$$
$$\text{So} \qquad (D^2 + 4D)(At^2 + Bt + C)$$

The ridiculously fabricated example problem

- Notice that

$$\begin{aligned} & D(At^2 + Bt + C) = (2At + B) \\ & D^2(At^2 + Bt + C) = 2A. \\ \text{So} \quad & (D^2 + 4D)(At^2 + Bt + C) \\ = \quad & D^2(At^2 + Bt + C) + 4D(At^2 + Bt + C) \end{aligned}$$

The ridiculously fabricated example problem

- Notice that

$$D(At^2 + Bt + C) = (2At + B)$$
$$D^2(At^2 + Bt + C) = 2A.$$

So
$$\begin{aligned} & (D^2 + 4D)(At^2 + Bt + C) \\ = \; & D^2(At^2 + Bt + C) + 4D(At^2 + Bt + C) \\ = \; & 2A + 4(2At + B) \end{aligned}$$

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(At^2 + Bt + C) = (2At + B) \\
& D^2(At^2 + Bt + C) = 2A. \\
\text{So} \quad & (D^2 + 4D)(At^2 + Bt + C) \\
= \ & D^2(At^2 + Bt + C) + 4D(At^2 + Bt + C) \\
= \ & 2A + 4(2At + B) = 8At + 2A + 4B.
\end{aligned}
$$

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(At^2+Bt+C) = (2At+B) \\
& D^2(At^2+Bt+C) = 2A. \\
\text{So} \quad & (D^2+4D)(At^2+Bt+C) \\
= \quad & D^2(At^2+Bt+C) + 4D(At^2+Bt+C) \\
= \quad & 2A + 4(2At+B) = 8At + 2A + 4B. \\
\text{Therefore} \quad & P'' + 2P' - 3P = e^t(8At + 2A + 4B).
\end{aligned}
$$

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(At^2 + Bt + C) = (2At + B) \\
& D^2(At^2 + Bt + C) = 2A. \\
\text{So} \quad & (D^2 + 4D)(At^2 + Bt + C) \\
= \quad & D^2(At^2 + Bt + C) + 4D(At^2 + Bt + C) \\
= \quad & 2A + 4(2At + B) = 8At + 2A + 4B. \\
\text{Therefore} \quad & P'' + 2P' - 3P = e^t(8At + 2A + 4B).
\end{aligned}
$$

- But the right hand side we have is $e^t(t^2 + 4)$.

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(At^2+Bt+C) = (2At+B) \\
& D^2(At^2+Bt+C) = 2A. \\
\text{So} \quad & (D^2+4D)(At^2+Bt+C) \\
= \quad & D^2(At^2+Bt+C) + 4D(At^2+Bt+C) \\
= \quad & 2A + 4(2At+B) = 8At + 2A + 4B. \\
\text{Therefore} \quad & P'' + 2P' - 3P = e^t(8At + 2A + 4B).
\end{aligned}
$$

- But the right hand side we have is $e^t(t^2+4)$. There is no term corresponds to t^2e^t

The ridiculously fabricated example problem

- Notice that

$$\begin{aligned} & D(At^2+Bt+C) = (2At+B) \\ & D^2(At^2+Bt+C) = 2A. \\ \text{So} \quad & (D^2+4D)(At^2+Bt+C) \\ = \; & D^2(At^2+Bt+C) + 4D(At^2+Bt+C) \\ = \; & 2A + 4(2At+B) = 8At + 2A + 4B. \\ \text{Therefore} \quad & P'' + 2P' - 3P = e^t(8At+2A+4B). \end{aligned}$$

- But the right hand side we have is $e^t(t^2+4)$. There is no term corresponds to t^2e^t so our first try fails.

The ridiculously fabricated example problem

- Notice that

$$\begin{aligned}
& D(At^2+Bt+C) = (2At+B) \\
& D^2(At^2+Bt+C) = 2A. \\
\text{So} \quad & (D^2+4D)(At^2+Bt+C) \\
= \quad & D^2(At^2+Bt+C) + 4D(At^2+Bt+C) \\
= \quad & 2A + 4(2At+B) = 8At + 2A + 4B. \\
\text{Therefore} \quad & P'' + 2P' - 3P = e^t(8At+2A+4B).
\end{aligned}$$

- But the right hand side we have is $e^t(t^2+4)$. There is no term corresponds to t^2e^t so our first try fails.
- Remark:

The ridiculously fabricated example problem

- Notice that

$$\begin{aligned} & D(At^2+Bt+C) = (2At+B) \\ & D^2(At^2+Bt+C) = 2A. \\ \text{So} \quad & (D^2+4D)(At^2+Bt+C) \\ = \quad & D^2(At^2+Bt+C) + 4D(At^2+Bt+C) \\ = \quad & 2A + 4(2At+B) = 8At + 2A + 4B. \\ \text{Therefore} \quad & P'' + 2P' - 3P = e^t(8At+2A+4B). \end{aligned}$$

- But the right hand side we have is $e^t(t^2+4)$. There is no term corresponds to t^2e^t so our first try fails.
- Remark: The first try always fails whenever your $e^{\alpha t}$ corresponds to part of the complementary solutions

The ridiculously fabricated example problem

- Notice that

$$\begin{aligned}
& D(At^2+Bt+C) = (2At+B) \\
& D^2(At^2+Bt+C) = 2A. \\
\text{So} \quad & (D^2+4D)(At^2+Bt+C) \\
= \quad & D^2(At^2+Bt+C) + 4D(At^2+Bt+C) \\
= \quad & 2A + 4(2At+B) = 8At + 2A + 4B. \\
\text{Therefore} \quad & P'' + 2P' - 3P = e^t(8At + 2A + 4B).
\end{aligned}$$

- But the right hand side we have is $e^t(t^2+4)$. There is no term corresponds to t^2e^t so our first try fails.
- Remark: The first try always fails whenever your $e^{\alpha t}$ corresponds to part of the complementary solutions (in this case e^t).

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(At^2+Bt+C) = (2At+B) \\
& D^2(At^2+Bt+C) = 2A. \\
\text{So} \quad & (D^2+4D)(At^2+Bt+C) \\
= \quad & D^2(At^2+Bt+C) + 4D(At^2+Bt+C) \\
= \quad & 2A + 4(2At+B) = 8At + 2A + 4B. \\
\text{Therefore} \quad & P'' + 2P' - 3P = e^t(8At + 2A + 4B).
\end{aligned}
$$

- But the right hand side we have is $e^t(t^2+4)$. There is no term corresponds to t^2e^t so our first try fails.
- Remark: The first try always fails whenever your $e^{\alpha t}$ corresponds to part of the complementary solutions (in this case e^t). And it is very easy to see this from the above arguments:

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(At^2+Bt+C) = (2At+B) \\
& D^2(At^2+Bt+C) = 2A. \\
\text{So} \quad & (D^2+4D)(At^2+Bt+C) \\
= \quad & D^2(At^2+Bt+C) + 4D(At^2+Bt+C) \\
= \quad & 2A + 4(2At+B) = 8At + 2A + 4B. \\
\text{Therefore} \quad & P'' + 2P' - 3P = e^t(8At+2A+4B).
\end{aligned}
$$

- But the right hand side we have is $e^t(t^2+4)$. There is no term corresponds to t^2e^t so our first try fails.
- Remark: The first try always fails whenever your $e^{\alpha t}$ corresponds to part of the complementary solutions (in this case e^t). And it is very easy to see this from the above arguments: the $f(D+1)$ above DOES NOT HAVE A CONSTANT TERM!

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(At^2+Bt+C)=(2At+B)\\
& D^2(At^2+Bt+C)=2A.\\
\text{So}\quad & (D^2+4D)(At^2+Bt+C)\\
= \quad & D^2(At^2+Bt+C)+4D(At^2+Bt+C)\\
= \quad & 2A+4(2At+B)=8At+2A+4B.\\
\text{Therefore}\quad & P''+2P'-3P=e^t(8At+2A+4B).
\end{aligned}
$$

- But the right hand side we have is $e^t(t^2+4)$. There is no term corresponds to t^2e^t so our first try fails.
- Remark: The first try always fails whenever your $e^{\alpha t}$ corresponds to part of the complementary solutions (in this case e^t). And it is very easy to see this from the above arguments: the $f(D+1)$ above DOES NOT HAVE A CONSTANT TERM! Think about why this leads to the failure of first try.

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$.

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x - 1)(x + 3)$):

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$P'' + 2P' - 3P \quad =$$

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$P'' + 2P' - 3P = (D^2 + 2D - 3)P = f(D)P$$

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$\begin{aligned} P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\ &= f(D)(e^t(At^3 + Bt^2 + Ct)) \end{aligned}$$

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$\begin{aligned} P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\ &= f(D)(e^t(At^3 + Bt^2 + Ct)) \\ &= e^t f(D+1)(At^3 + Bt^2 + Ct) \end{aligned}$$

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$
\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)(e^t(At^3 + Bt^2 + Ct)) \\
&= e^t f(D+1)(At^3 + Bt^2 + Ct) \\
&= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct).
\end{aligned}
$$

The ridiculously fabricated example problem

- Let's do the second try: $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$\begin{aligned} P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\ &= f(D)(e^t(At^3 + Bt^2 + Ct)) \\ &= e^t f(D+1)(At^3 + Bt^2 + Ct) \\ &= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct). \\ \text{Notice that} \quad & D(At^3 + Bt^2 + Ct) = 3At^2 + 2Bt + C, \end{aligned}$$

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$\begin{aligned} P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\ &= f(D)(e^t(At^3 + Bt^2 + Ct)) \\ &= e^t f(D+1)(At^3 + Bt^2 + Ct) \\ &= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct). \\ \text{Notice that} \quad & D(At^3 + Bt^2 + Ct) = 3At^2 + 2Bt + C, \\ & D^2(At^3 + Bt^2 + Ct) = 6At + 2B. \end{aligned}$$

The ridiculously fabricated example problem

- Let's do the second try: $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)(e^t(At^3 + Bt^2 + Ct)) \\
&= e^t f(D+1)(At^3 + Bt^2 + Ct) \\
&= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct). \\
\text{Notice that} \quad & D(At^3 + Bt^2 + Ct) = 3At^2 + 2Bt + C, \\
& D^2(At^3 + Bt^2 + Ct) = 6At + 2B. \\
\text{Then } P'' + 2P' - 3P &=
\end{aligned}$$

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)(e^t(At^3 + Bt^2 + Ct)) \\
&= e^t f(D+1)(At^3 + Bt^2 + Ct) \\
&= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct). \\
\text{Notice that} \quad & D(At^3 + Bt^2 + Ct) = 3At^2 + 2Bt + C, \\
& D^2(At^3 + Bt^2 + Ct) = 6At + 2B. \\
\text{Then } P'' + 2P' - 3P &= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct)
\end{aligned}$$

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$
\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)(e^t(At^3 + Bt^2 + Ct)) \\
&= e^t f(D+1)(At^3 + Bt^2 + Ct) \\
&= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct). \\
\text{Notice that} \quad & D(At^3 + Bt^2 + Ct) = 3At^2 + 2Bt + C, \\
& D^2(At^3 + Bt^2 + Ct) = 6At + 2B. \\
\text{Then } P'' + 2P' - 3P &= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct) \\
&= e^t(6At + 2B
\end{aligned}
$$

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)(e^t(At^3 + Bt^2 + Ct)) \\
&= e^t f(D+1)(At^3 + Bt^2 + Ct) \\
&= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct). \\
\text{Notice that} \quad & D(At^3 + Bt^2 + Ct) = 3At^2 + 2Bt + C, \\
& D^2(At^3 + Bt^2 + Ct) = 6At + 2B. \\
\text{Then } P'' + 2P' - 3P &= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct) \\
&= e^t(6At + 2B + 4(3At^2 + 2Bt + C))
\end{aligned}$$

The ridiculously fabricated example problem

- Let's do the second try:
  $P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct)$. Again we use the exponential-shift rule to compute $P'' + 2P' - 3P$ (Recall that $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$
\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)(e^t(At^3 + Bt^2 + Ct)) \\
&= e^t f(D+1)(At^3 + Bt^2 + Ct) \\
&= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct). \\
\text{Notice that} \quad & D(At^3 + Bt^2 + Ct) = 3At^2 + 2Bt + C, \\
& D^2(At^3 + Bt^2 + Ct) = 6At + 2B. \\
\text{Then } P'' + 2P' - 3P &= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct) \\
&= e^t(6At + 2B + 4(3At^2 + 2Bt + C)) \\
&= e^t(12A^2 + (6A + 8B)t + 2B + 4C)
\end{aligned}
$$

The ridiculously fabricated example problem

- Now compare the coefficients:

The ridiculously fabricated example problem

- Now compare the coefficients:

$$e^t(12A^2 + (6A + 8B)t + 2B + 4C) = e^t(t^2 + 4)$$

The ridiculously fabricated example problem

- Now compare the coefficients:

$$
\begin{aligned}
& e^t(12A^2 + (6A + 8B)t + 2B + 4C) = e^t(t^2 + 4) \\
\Rightarrow \quad & 12A = 1, 6A + 8B = 0, 2B + 4C = 4
\end{aligned}
$$

The ridiculously fabricated example problem

- Now compare the coefficients:

$$\begin{aligned} & e^t(12A^2 + (6A + 8B)t + 2B + 4C) = e^t(t^2 + 4) \\ \Rightarrow\quad & 12A = 1, 6A + 8B = 0, 2B + 4C = 4 \\ \Rightarrow\quad & A = \frac{1}{12}, \end{aligned}$$

The ridiculously fabricated example problem

- Now compare the coefficients:

$$\begin{aligned} & e^t(12A^2 + (6A + 8B)t + 2B + 4C) = e^t(t^2 + 4) \\ \Rightarrow \quad & 12A = 1, 6A + 8B = 0, 2B + 4C = 4 \\ \Rightarrow \quad & A = \frac{1}{12}, B = -\frac{6}{8} \cdot \frac{1}{12} \end{aligned}$$

The ridiculously fabricated example problem

- Now compare the coefficients:

$$
\begin{aligned}
& e^t(12A^2 + (6A + 8B)t + 2B + 4C) = e^t(t^2 + 4) \\
\Rightarrow \quad & 12A = 1, 6A + 8B = 0, 2B + 4C = 4 \\
\Rightarrow \quad & A = \frac{1}{12}, B = -\frac{6}{8} \cdot \frac{1}{12} = -\frac{1}{16},
\end{aligned}
$$

The ridiculously fabricated example problem

- Now compare the coefficients:

$$
\begin{aligned}
& e^t(12A^2 + (6A + 8B)t + 2B + 4C) = e^t(t^2 + 4) \\
\Rightarrow\quad & 12A = 1, 6A + 8B = 0, 2B + 4C = 4 \\
\Rightarrow\quad & A = \frac{1}{12}, B = -\frac{6}{8} \cdot \frac{1}{12} = -\frac{1}{16}, C = \frac{1}{4}(4 - 2B)
\end{aligned}
$$

The ridiculously fabricated example problem

- Now compare the coefficients:

$$
\begin{aligned}
& e^t(12A^2 + (6A + 8B)t + 2B + 4C) = e^t(t^2 + 4) \\
\Rightarrow \quad & 12A = 1, 6A + 8B = 0, 2B + 4C = 4 \\
\Rightarrow \quad & A = \frac{1}{12}, B = -\frac{6}{8} \cdot \frac{1}{12} = -\frac{1}{16}, C = \frac{1}{4}(4 - 2B) = 1 - \frac{B}{2}
\end{aligned}
$$

The ridiculously fabricated example problem

- Now compare the coefficients:

$$
\begin{aligned}
& e^t(12A^2 + (6A + 8B)t + 2B + 4C) = e^t(t^2 + 4) \\
\Rightarrow \quad & 12A = 1, 6A + 8B = 0, 2B + 4C = 4 \\
\Rightarrow \quad & A = \frac{1}{12}, B = -\frac{6}{8} \cdot \frac{1}{12} = -\frac{1}{16}, C = \frac{1}{4}(4 - 2B) = 1 - \frac{B}{2} = \frac{33}{32}
\end{aligned}
$$

The ridiculously fabricated example problem

- Now compare the coefficients:

$$
\begin{aligned}
& e^t(12A^2 + (6A+8B)t + 2B + 4C) = e^t(t^2+4) \\
\Rightarrow\ & 12A = 1, 6A + 8B = 0, 2B + 4C = 4 \\
\Rightarrow\ & A = \frac{1}{12}, B = -\frac{6}{8}\cdot\frac{1}{12} = -\frac{1}{16}, C = \frac{1}{4}(4-2B) = 1 - \frac{B}{2} = \frac{33}{32} \\
\Rightarrow\ & P(t) = (\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)e^t.
\end{aligned}
$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION!

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$P'' + 2P' - 3P \quad =$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$P'' + 2P' - 3P = (D^2 + 2D - 3)P = f(D)P$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$\begin{aligned} P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\ &= f(D)[e^t(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)] \end{aligned}$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$\begin{aligned} P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\ &= f(D)[e^t(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)] \\ &= e^t f(D+1)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \end{aligned}$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$\begin{aligned} P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\ &= f(D)[e^t(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)] \\ &= e^t f(D+1)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\ &= e^t(D^2 + 4D)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \end{aligned}$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$
\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)[e^t(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)] \\
&= e^t f(D+1)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(D^2 + 4D)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(4(\frac{3}{12}t^2 - \frac{2}{16}t + \frac{33}{32})
\end{aligned}
$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)[e^t(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)] \\
&= e^t f(D+1)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t (D^2 + 4D)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t (4(\frac{3}{12}t^2 - \frac{2}{16}t + \frac{33}{32}) + \frac{2 \times 3}{12}t - \frac{2}{16})
\end{aligned}$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)[e^t(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)] \\
&= e^t f(D+1)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(D^2 + 4D)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(4(\frac{3}{12}t^2 - \frac{2}{16}t + \frac{33}{32}) + \frac{2\times 3}{12}t - \frac{2}{16}) \\
&= e^t(t^2 - \frac{1}{2}t + \frac{33}{8}
\end{aligned}$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)[e^t(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)] \\
&= e^t f(D+1)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(D^2 + 4D)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(4(\frac{3}{12}t^2 - \frac{2}{16}t + \frac{33}{32}) + \frac{2 \times 3}{12}t - \frac{2}{16}) \\
&= e^t(t^2 - \frac{1}{2}t + \frac{33}{8} + \frac{1}{2}t - \frac{1}{8})
\end{aligned}$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)[e^t(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)] \\
&= e^t f(D+1)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(D^2 + 4D)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(4(\frac{3}{12}t^2 - \frac{2}{16}t + \frac{33}{32}) + \frac{2\times 3}{12}t - \frac{2}{16}) \\
&= e^t(t^2 - \frac{1}{2}t + \frac{33}{8} + \frac{1}{2}t - \frac{1}{8}) \\
&= e^t(t^2 + 4)
\end{aligned}$$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$
\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)[e^t(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)] \\
&= e^t f(D+1)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(D^2 + 4D)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(4(\frac{3}{12}t^2 - \frac{2}{16}t + \frac{33}{32}) + \frac{2 \times 3}{12}t - \frac{2}{16}) \\
&= e^t(t^2 - \frac{1}{2}t + \frac{33}{8} + \frac{1}{2}t - \frac{1}{8}) \\
&= e^t(t^2 + 4)
\end{aligned}
$$

So $P(t)$ is a solution of $y'' + 2y' - 3y = e^t(t^2 + 4)$

The ridiculously fabricated example problem

- CHECK YOUR SOLUTION! Again use the exponential shift rule

$$\begin{aligned}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)[e^t(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)] \\
&= e^t f(D+1)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(D^2 + 4D)(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t) \\
&= e^t(4(\frac{3}{12}t^2 - \frac{2}{16}t + \frac{33}{32}) + \frac{2 \times 3}{12}t - \frac{2}{16}) \\
&= e^t(t^2 - \frac{1}{2}t + \frac{33}{8} + \frac{1}{2}t - \frac{1}{8}) \\
&= e^t(t^2 + 4)
\end{aligned}$$

So $P(t)$ is a solution of $y'' + 2y' - 3y = e^t(t^2 + 4)$ and the first part of the solution is done.

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2+4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part.

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part. Some redundant steps will be skipped in the following computations.

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part. Some redundant steps will be skipped in the following computations.

- Let $P(t)$ be a particular solution of

$$y'' + 2y' - 3y = e^{-3t}\cos 3t.$$

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part. Some redundant steps will be skipped in the following computations.

- Let $P(t)$ be a particular solution of

  $$y'' + 2y' - 3y = e^{-3t}\cos 3t.$$

  In this case the template is going to be

  $$P(t) = e^{-3t}(A\cos 3t + B\sin 3t).$$

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part. Some redundant steps will be skipped in the following computations.

- Let $P(t)$ be a particular solution of

  $$y'' + 2y' - 3y = e^{-3t}\cos 3t.$$

  In this case the template is going to be

  $$P(t) = e^{-3t}(A\cos 3t + B\sin 3t).$$

- Find $P'' + 2P' - 3P$

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part. Some redundant steps will be skipped in the following computations.

- Let $P(t)$ be a particular solution of

  $$y'' + 2y' - 3y = e^{-3t}\cos 3t.$$

  In this case the template is going to be

  $$P(t) = e^{-3t}(A\cos 3t + B\sin 3t).$$

- Find $P'' + 2P' - 3P$ (Again $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2+4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part. Some redundant steps will be skipped in the following computations.

- Let $P(t)$ be a particular solution of

  $$y'' + 2y' - 3y = e^{-3t}\cos 3t.$$

  In this case the template is going to be

  $$P(t) = e^{-3t}(A\cos 3t + B\sin 3t).$$

- Find $P'' + 2P' - 3P$ (Again $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

  $$P'' + 2P' - 3P \quad =$$

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part. Some redundant steps will be skipped in the following computations.

- Let $P(t)$ be a particular solution of

$$y'' + 2y' - 3y = e^{-3t}\cos 3t.$$

  In this case the template is going to be

$$P(t) = e^{-3t}(A\cos 3t + B\sin 3t).$$

- Find $P'' + 2P' - 3P$ (Again $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$P'' + 2P' - 3P \quad = \quad f(D)(e^{-3t}(A\cos 3t + B\sin 3t)$$

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2+4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part. Some redundant steps will be skipped in the following computations.

- Let $P(t)$ be a particular solution of

  $$y'' + 2y' - 3y = e^{-3t}\cos 3t.$$

  In this case the template is going to be

  $$P(t) = e^{-3t}(A\cos 3t + B\sin 3t).$$

- Find $P'' + 2P' - 3P$ (Again $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

  $$\begin{aligned} P'' + 2P' - 3P &= f(D)(e^{-3t}(A\cos 3t + B\sin 3t) \\ &= e^{-3t} f(D-3)(A\cos 3t + B\sin 3t) \end{aligned}$$

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part. Some redundant steps will be skipped in the following computations.

- Let $P(t)$ be a particular solution of

  $$y'' + 2y' - 3y = e^{-3t}\cos 3t.$$

  In this case the template is going to be

  $$P(t) = e^{-3t}(A\cos 3t + B\sin 3t).$$

- Find $P'' + 2P' - 3P$ (Again $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$\begin{aligned} P'' + 2P' - 3P &= f(D)(e^{-3t}(A\cos 3t + B\sin 3t) \\ &= e^{-3t}f(D-3)(A\cos 3t + B\sin 3t) \\ &= e^{-3t}(D-3-1)(D-3+3)(A\cos 3t + B\sin 3t) \end{aligned}$$

The ridiculously fabricated example problem

Recall that our ODE is

$$y'' + 2y' - 3y = e^t(t^2+4) + e^{-3t}\cos 3t + \cos 4t + t^2,$$

We now look at the second part. Some redundant steps will be skipped in the following computations.

- Let $P(t)$ be a particular solution of

  $$y'' + 2y' - 3y = e^{-3t}\cos 3t.$$

  In this case the template is going to be

  $$P(t) = e^{-3t}(A\cos 3t + B\sin 3t).$$

- Find $P'' + 2P' - 3P$ (Again $f(x) = x^2 + 2x - 3 = (x-1)(x+3)$):

$$\begin{aligned} P'' + 2P' - 3P &= f(D)(e^{-3t}(A\cos 3t + B\sin 3t) \\ &= e^{-3t}f(D-3)(A\cos 3t + B\sin 3t) \\ &= e^{-3t}(D-3-1)(D-3+3)(A\cos 3t + B\sin 3t) \\ &= e^{-3t}(D^2-4D)(A\cos 3t + B\sin 3t). \end{aligned}$$

The ridiculously fabricated example problem

- Notice that

The ridiculously fabricated example problem

- Notice that

$$D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t,$$

The ridiculously fabricated example problem

- Notice that

$$D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t,$$
$$D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t.$$

The ridiculously fabricated example problem

- Notice that

$$D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t,$$
$$D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t.$$

Then $\quad (D^2 - 4D)(A\cos 3t + B\sin 3t)$

The ridiculously fabricated example problem

- Notice that

$$D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t,$$
$$D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t.$$

Then

$$\begin{aligned} & (D^2 - 4D)(A\cos 3t + B\sin 3t) \\ = \; & -9A\cos 3t - 9B\sin 3t \end{aligned}$$

The ridiculously fabricated example problem

- Notice that

$$D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t,$$
$$D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t.$$

Then

$$(D^2 - 4D)(A\cos 3t + B\sin 3t)$$
$$= -9A\cos 3t - 9B\sin 3t + 12A\sin 3t - 12B\cos 3t$$

The ridiculously fabricated example problem

- Notice that

$$D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t,$$
$$D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t.$$

Then

$$\begin{aligned} & (D^2 - 4D)(A\cos 3t + B\sin 3t) \\ = \; & -9A\cos 3t - 9B\sin 3t + 12A\sin 3t - 12B\cos 3t \\ = \; & (12A - 9B)\sin 3t + (-9A - 12B)\cos 3t. \end{aligned}$$

The ridiculously fabricated example problem

- Notice that

$$\begin{aligned} & D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t, \\ & D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t. \end{aligned}$$

Then

$$\begin{aligned} & (D^2 - 4D)(A\cos 3t + B\sin 3t) \\ = \; & -9A\cos 3t - 9B\sin 3t + 12A\sin 3t - 12B\cos 3t \\ = \; & (12A - 9B)\sin 3t + (-9A - 12B)\cos 3t. \end{aligned}$$

Thus

$$P'' + 2P' - 3P$$

The ridiculously fabricated example problem

- Notice that

$$D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t,$$

$$D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t.$$

Then $(D^2 - 4D)(A\cos 3t + B\sin 3t)$

$$= -9A\cos 3t - 9B\sin 3t + 12A\sin 3t - 12B\cos 3t$$

$$= (12A - 9B)\sin 3t + (-9A - 12B)\cos 3t.$$

Thus $P'' + 2P' - 3P = e^{-3t}(D^2 - 4D)(A\cos 3t + B\sin 3t)$

The ridiculously fabricated example problem

- Notice that

$$D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t,$$
$$D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t.$$

Then

$$\begin{aligned} & (D^2 - 4D)(A\cos 3t + B\sin 3t) \\ = \; & -9A\cos 3t - 9B\sin 3t + 12A\sin 3t - 12B\cos 3t \\ = \; & (12A - 9B)\sin 3t + (-9A - 12B)\cos 3t. \end{aligned}$$

Thus

$$\begin{aligned} & P'' + 2P' - 3P = e^{-3t}(D^2 - 4D)(A\cos 3t + B\sin 3t) \\ = \; & e^{-3t}[(12A - 9B)\sin 3t + (-9A - 12B)\cos 3t] \end{aligned}$$

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t, \\
& D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t. \\
\text{Then} \quad & (D^2 - 4D)(A\cos 3t + B\sin 3t) \\
= \quad & -9A\cos 3t - 9B\sin 3t + 12A\sin 3t - 12B\cos 3t \\
= \quad & (12A - 9B)\sin 3t + (-9A - 12B)\cos 3t. \\
\text{Thus} \quad & P'' + 2P' - 3P = e^{-3t}(D^2 - 4D)(A\cos 3t + B\sin 3t) \\
= \quad & e^{-3t}[(12A - 9B)\sin 3t + (-9A - 12B)\cos 3t]
\end{aligned}
$$

- Now compare the coefficients:

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t, \\
& D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t. \\
\text{Then} \quad & (D^2 - 4D)(A\cos 3t + B\sin 3t) \\
= \quad & -9A\cos 3t - 9B\sin 3t + 12A\sin 3t - 12B\cos 3t \\
= \quad & (12A - 9B)\sin 3t + (-9A - 12B)\cos 3t. \\
\text{Thus} \quad & P'' + 2P' - 3P = e^{-3t}(D^2 - 4D)(A\cos 3t + B\sin 3t) \\
= \quad & e^{-3t}[(12A - 9B)\sin 3t + (-9A - 12B)\cos 3t]
\end{aligned}
$$

- Now compare the coefficients:

$$e^{-3t}[(12A - 9B)\sin 3t + (-9A - 12B)\cos 3t] = e^{-3t}\cos 3t$$

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t, \\
& D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t.
\end{aligned}
$$

$$
\begin{aligned}
\text{Then} \quad & (D^2 - 4D)(A\cos 3t + B\sin 3t) \\
= \quad & -9A\cos 3t - 9B\sin 3t + 12A\sin 3t - 12B\cos 3t \\
= \quad & (12A - 9B)\sin 3t + (-9A - 12B)\cos 3t. \\
\text{Thus} \quad & P'' + 2P' - 3P = e^{-3t}(D^2 - 4D)(A\cos 3t + B\sin 3t) \\
= \quad & e^{-3t}[(12A - 9B)\sin 3t + (-9A - 12B)\cos 3t]
\end{aligned}
$$

- Now compare the coefficients:

$$
\begin{aligned}
& e^{-3t}[(12A - 9B)\sin 3t + (-9A - 12B)\cos 3t] = e^{-3t}\cos 3t \\
\Rightarrow \quad & 12A - 9B = 0,
\end{aligned}
$$

The ridiculously fabricated example problem

- Notice that

$$D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t,$$
$$D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t.$$

$$\begin{aligned} \text{Then} \quad & (D^2 - 4D)(A\cos 3t + B\sin 3t) \\ = \quad & -9A\cos 3t - 9B\sin 3t + 12A\sin 3t - 12B\cos 3t \\ = \quad & (12A - 9B)\sin 3t + (-9A - 12B)\cos 3t. \\ \text{Thus} \quad & P'' + 2P' - 3P = e^{-3t}(D^2 - 4D)(A\cos 3t + B\sin 3t) \\ = \quad & e^{-3t}[(12A - 9B)\sin 3t + (-9A - 12B)\cos 3t] \end{aligned}$$

- Now compare the coefficients:

$$e^{-3t}[(12A - 9B)\sin 3t + (-9A - 12B)\cos 3t] = e^{-3t}\cos 3t$$
$$\Rightarrow \quad 12A - 9B = 0, \; -9A - 12B = 1,$$

The ridiculously fabricated example problem

- Notice that

$$
\begin{aligned}
& D(A\cos 3t + B\sin 3t) = -3A\sin 3t + 3B\cos 3t, \\
& D^2(A\cos 3t + B\sin 3t) = -9A\cos 3t - 9B\sin 3t. \\
\text{Then} \quad & (D^2 - 4D)(A\cos 3t + B\sin 3t) \\
= \quad & -9A\cos 3t - 9B\sin 3t + 12A\sin 3t - 12B\cos 3t \\
= \quad & (12A - 9B)\sin 3t + (-9A - 12B)\cos 3t. \\
\text{Thus} \quad & P'' + 2P' - 3P = e^{-3t}(D^2 - 4D)(A\cos 3t + B\sin 3t) \\
= \quad & e^{-3t}[(12A - 9B)\sin 3t + (-9A - 12B)\cos 3t]
\end{aligned}
$$

- Now compare the coefficients:

$$
\begin{aligned}
& e^{-3t}[(12A - 9B)\sin 3t + (-9A - 12B)\cos 3t] = e^{-3t}\cos 3t \\
\Rightarrow \quad & 12A - 9B = 0, \ -9A - 12B = 1, \\
\Rightarrow \quad & A = -1/25, B = -4/75
\end{aligned}
$$

The ridiculously fabricated example problem

- So our particular solution would then be

The ridiculously fabricated example problem

- So our particular solution would then be

$$P(t) = e^{-3t}\left(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t\right).$$

The ridiculously fabricated example problem

- So our particular solution would then be

$$P(t) = e^{-3t}(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t).$$

- CHECK!

$$P'' + 2P' - 3P$$

The ridiculously fabricated example problem

- So our particular solution would then be

$$P(t) = e^{-3t}\left(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t\right).$$

- CHECK!

$$P'' + 2P' - 3P \quad = \quad e^{-3t}(D^2 - 4D)\left(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t\right)$$

The ridiculously fabricated example problem

- So our particular solution would then be

$$P(t) = e^{-3t}(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t).$$

- CHECK!

$$\begin{aligned} P'' + 2P' - 3P &= e^{-3t}(D^2 - 4D)(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t) \\ &= e^{-3t}(\frac{4}{25}(-3\sin 3t) + \frac{16}{75}(3\cos 3t) \end{aligned}$$

The ridiculously fabricated example problem

- So our particular solution would then be

$$P(t) = e^{-3t}(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t).$$

- CHECK!

$$\begin{aligned} P'' + 2P' - 3P &= e^{-3t}(D^2 - 4D)(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t) \\ &= e^{-3t}(\frac{4}{25}(-3\sin 3t) + \frac{16}{75}(3\cos 3t) \\ &+ \frac{1}{25}(9\cos 3t) + \frac{4}{75}9\sin 35) \end{aligned}$$

The ridiculously fabricated example problem

- So our particular solution would then be

$$P(t) = e^{-3t}(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t).$$

- CHECK!

$$\begin{aligned} P'' + 2P' - 3P &= e^{-3t}(D^2 - 4D)(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t) \\ &= e^{-3t}(\frac{4}{25}(-3\sin 3t) + \frac{16}{75}(3\cos 3t) \\ &+ \frac{1}{25}(9\cos 3t) + \frac{4}{75}9\sin 35) \\ &= e^{-3t}(\frac{16+9}{25}\cos 3t + \frac{12 - 4\times 3}{25}\sin 3t) \end{aligned}$$

The ridiculously fabricated example problem

- So our particular solution would then be

$$P(t) = e^{-3t}(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t).$$

- CHECK!

$$\begin{aligned} P'' + 2P' - 3P &= e^{-3t}(D^2 - 4D)(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t) \\ &= e^{-3t}(\frac{4}{25}(-3\sin 3t) + \frac{16}{75}(3\cos 3t) \\ &+ \frac{1}{25}(9\cos 3t) + \frac{4}{75}9\sin 35) \\ &= e^{-3t}(\frac{16+9}{25}\cos 3t + \frac{12 - 4\times 3}{25}\sin 3t) \\ &= e^{-3t}\cos 3t \end{aligned}$$

The ridiculously fabricated example problem

- So our particular solution would then be

$$P(t) = e^{-3t}\left(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t\right).$$

- CHECK!

$$\begin{aligned}
P'' + 2P' - 3P &= e^{-3t}(D^2 - 4D)\left(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t\right) \\
&= e^{-3t}\left(\frac{4}{25}(-3\sin 3t) + \frac{16}{75}(3\cos 3t)\right. \\
&+ \left.\frac{1}{25}(9\cos 3t) + \frac{4}{75}9\sin 35\right) \\
&= e^{-3t}\left(\frac{16+9}{25}\cos 3t + \frac{12 - 4\times 3}{25}\sin 3t\right) \\
&= e^{-3t}\cos 3t
\end{aligned}$$

So it is a particular solution.

The ridiculously fabricated example problem

- Remark:

The ridiculously fabricated example problem

- Remark: Why first try did not fail even when you have a e^{-3t} in this case?

The ridiculously fabricated example problem

- Remark: Why first try did not fail even when you have a e^{-3t} in this case? If you have understood MIT Lecture 13 well enough,

The ridiculously fabricated example problem

- Remark: Why first try did not fail even when you have a e^{-3t} in this case? If you have understood MIT Lecture 13 well enough, $e^{-3t}\cos 3t$ corresponds to the characteristic root $-3+3i$.

The ridiculously fabricated example problem

- Remark: Why first try did not fail even when you have a e^{-3t} in this case? If you have understood MIT Lecture 13 well enough, $e^{-3t}\cos 3t$ corresponds to the characteristic root $-3+3i$. In other words, if the ODE looks like

$$y'' - 6y + 18 = e^{-3t}\cos 3t.$$

The ridiculously fabricated example problem

- Remark: Why first try did not fail even when you have a e^{-3t} in this case? If you have understood MIT Lecture 13 well enough, $e^{-3t}\cos 3t$ corresponds to the characteristic root $-3+3i$. In other words, if the ODE looks like

$$y'' - 6y + 18 = e^{-3t}\cos 3t.$$

then your first try should fail.
- Remark:

The ridiculously fabricated example problem

- Remark: Why first try did not fail even when you have a e^{-3t} in this case? If you have understood MIT Lecture 13 well enough, $e^{-3t}\cos 3t$ corresponds to the characteristic root $-3+3i$. In other words, if the ODE looks like

$$y'' - 6y + 18 = e^{-3t}\cos 3t.$$

  then your first try should fail.
- Remark: From here you can also conclude that for second order linear homogeneous ODE,

The ridiculously fabricated example problem

- Remark: Why first try did not fail even when you have a e^{-3t} in this case? If you have understood MIT Lecture 13 well enough, $e^{-3t}\cos 3t$ corresponds to the characteristic root $-3+3i$. In other words, if the ODE looks like

$$y'' - 6y + 18 = e^{-3t}\cos 3t.$$

then your first try should fail.

- Remark: From here you can also conclude that for second order linear homogeneous ODE, at most you need to make a second try.

The ridiculously fabricated example problem

- Remark: Why first try did not fail even when you have a e^{-3t} in this case? If you have understood MIT Lecture 13 well enough, $e^{-3t}\cos 3t$ corresponds to the characteristic root $-3+3i$. In other words, if the ODE looks like

$$y'' - 6y + 18 = e^{-3t}\cos 3t.$$

  then your first try should fail.
- Remark: From here you can also conclude that for second order linear homogeneous ODE, at most you need to make a second try. Think about why.

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$.

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here,

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

  $P'' + 2P' - 3P$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

$$P'' + 2P' - 3P = -16A\cos 4t - 16B\sin 4t$$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

$$\begin{aligned} P'' + 2P' - 3P &= -16A\cos 4t - 16B\sin 4t \\ &+ 2(-4A\sin 4t + 4B\cos 4t) \end{aligned}$$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

$$\begin{aligned} P'' + 2P' - 3P &= -16A\cos 4t - 16B\sin 4t \\ &+ 2(-4A\sin 4t + 4B\cos 4t) \\ &- 3(A\cos 4t + B\sin 4t) \end{aligned}$$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

$$\begin{aligned} P'' + 2P' - 3P &= -16A\cos 4t - 16B\sin 4t \\ &+ 2(-4A\sin 4t + 4B\cos 4t) \\ &- 3(A\cos 4t + B\sin 4t) \\ &= (-19A + 8B)\cos 4t + (-19B - 8A)\sin 4t. \end{aligned}$$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

$$\begin{aligned} P'' + 2P' - 3P &= -16A\cos 4t - 16B\sin 4t \\ &+ 2(-4A\sin 4t + 4B\cos 4t) \\ &- 3(A\cos 4t + B\sin 4t) \\ &= (-19A + 8B)\cos 4t + (-19B - 8A)\sin 4t. \end{aligned}$$

- Compare the coefficients one has

$$-19A + 8B = 1,$$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

$$\begin{aligned} P'' + 2P' - 3P &= -16A\cos 4t - 16B\sin 4t \\ &+ 2(-4A\sin 4t + 4B\cos 4t) \\ &- 3(A\cos 4t + B\sin 4t) \\ &= (-19A + 8B)\cos 4t + (-19B - 8A)\sin 4t. \end{aligned}$$

- Compare the coefficients one has

$$-19A + 8B = 1, \; -19B - 8A = 0,$$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

$$\begin{aligned} P'' + 2P' - 3P &= -16A\cos 4t - 16B\sin 4t \\ &+ 2(-4A\sin 4t + 4B\cos 4t) \\ &- 3(A\cos 4t + B\sin 4t) \\ &= (-19A + 8B)\cos 4t + (-19B - 8A)\sin 4t. \end{aligned}$$

- Compare the coefficients one has

$$\begin{aligned} & -19A + 8B = 1, \; -19B - 8A = 0, \\ \Rightarrow \quad & A = -19B/8, \end{aligned}$$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

$$\begin{aligned} P'' + 2P' - 3P &= -16A\cos 4t - 16B\sin 4t \\ &+ 2(-4A\sin 4t + 4B\cos 4t) \\ &- 3(A\cos 4t + B\sin 4t) \\ &= (-19A + 8B)\cos 4t + (-19B - 8A)\sin 4t. \end{aligned}$$

- Compare the coefficients one has

$$\begin{aligned} & -19A + 8B = 1, \ -19B - 8A = 0, \\ \Rightarrow \quad & A = -19B/8, (19^2 + 64)B = (361 + 64)B = 425B = 8, \end{aligned}$$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

$$\begin{aligned} P'' + 2P' - 3P &= -16A\cos 4t - 16B\sin 4t \\ &+ 2(-4A\sin 4t + 4B\cos 4t) \\ &- 3(A\cos 4t + B\sin 4t) \\ &= (-19A + 8B)\cos 4t + (-19B - 8A)\sin 4t. \end{aligned}$$

- Compare the coefficients one has

$$\begin{aligned} & -19A + 8B = 1, \; -19B - 8A = 0, \\ \Rightarrow \quad & A = -19B/8, (19^2 + 64)B = (361 + 64)B = 425B = 8, \\ \Rightarrow \quad & A = -19/425, \end{aligned}$$

The ridiculously fabricated example problem

Now let's look at the third term:

$$y'' + 2y' - 3y = \cos 4t$$

- The template is then $P(t) = A\cos 4t + B\sin 4t$. Since you don't have exponential here, it would be easy to compute directly:

$$\begin{aligned} P'' + 2P' - 3P &= -16A\cos 4t - 16B\sin 4t \\ &+ 2(-4A\sin 4t + 4B\cos 4t) \\ &- 3(A\cos 4t + B\sin 4t) \\ &= (-19A + 8B)\cos 4t + (-19B - 8A)\sin 4t. \end{aligned}$$

- Compare the coefficients one has

$$\begin{aligned} & -19A + 8B = 1, -19B - 8A = 0, \\ \Rightarrow\ & A = -19B/8, (19^2 + 64)B = (361 + 64)B = 425B = 8, \\ \Rightarrow\ & A = -19/425, B = 8/425. \end{aligned}$$

The ridiculously fabricated example problem

- So the particular solution is

$$P(t) = -\frac{19}{425}\cos 4t + \frac{8}{425}\sin 4t.$$

The ridiculously fabricated example problem

- So the particular solution is

$$P(t) = -\frac{19}{425}\cos 4t + \frac{8}{425}\sin 4t.$$

- CHECK YOUR SOLUTION (skip).

Finally let's look at the third term:

$$y'' + 2y' - 3y = t^2$$

The ridiculously fabricated example problem

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$.

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$P'' + 2P' - 3P$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$P'' + 2P' - 3P \quad = \quad 2A$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$P'' + 2P' - 3P = 2A + 2(2At + B)$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$P'' + 2P' - 3P = 2A + 2(2At + B) - 3(At^2 + Bt + C)$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$\begin{aligned} P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\ &= -3At^2 + (4A - 3B)t + 2A + 2B - 3C \end{aligned}$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$\begin{aligned} P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\ &= -3At^2 + (4A - 3B)t + 2A + 2B - 3C \end{aligned}$$

- Compare the coefficients one has

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$\begin{aligned} P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\ &= -3At^2 + (4A - 3B)t + 2A + 2B - 3C \end{aligned}$$

- Compare the coefficients one has

$$-3A = 1,$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$\begin{aligned} P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\ &= -3At^2 + (4A - 3B)t + 2A + 2B - 3C \end{aligned}$$

- Compare the coefficients one has

$$-3A = 1, 4A - 3B = 0,$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$\begin{aligned} P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\ &= -3At^2 + (4A - 3B)t + 2A + 2B - 3C \end{aligned}$$

- Compare the coefficients one has

$$-3A = 1, 4A - 3B = 0, 2A + 2B - 3C = 0$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$\begin{aligned} P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\ &= -3At^2 + (4A - 3B)t + 2A + 2B - 3C \end{aligned}$$

- Compare the coefficients one has

$$\begin{aligned} & -3A = 1, 4A - 3B = 0, 2A + 2B - 3C = 0 \\ \Rightarrow \quad & A = -1/3, \end{aligned}$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$\begin{aligned} P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\ &= -3At^2 + (4A - 3B)t + 2A + 2B - 3C \end{aligned}$$

- Compare the coefficients one has

$$\begin{aligned} & -3A = 1, 4A - 3B = 0, 2A + 2B - 3C = 0 \\ \Rightarrow \quad & A = -1/3, B = -4/9, \end{aligned}$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$\begin{aligned} P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\ &= -3At^2 + (4A - 3B)t + 2A + 2B - 3C \end{aligned}$$

- Compare the coefficients one has

$$\begin{aligned} & -3A = 1, 4A - 3B = 0, 2A + 2B - 3C = 0 \\ \Rightarrow \quad & A = -1/3, B = -4/9, C = (2A + 2B)/3 = -14/27. \end{aligned}$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$\begin{aligned} P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\ &= -3At^2 + (4A - 3B)t + 2A + 2B - 3C \end{aligned}$$

- Compare the coefficients one has

$$\begin{aligned} & -3A = 1, 4A - 3B = 0, 2A + 2B - 3C = 0 \\ \Rightarrow \quad & A = -1/3, B = -4/9, C = (2A + 2B)/3 = -14/27. \\ \Rightarrow \quad & P(t) = -\frac{1}{3}t^2 - \frac{4}{9}t - \frac{14}{27}. \end{aligned}$$

The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$\begin{aligned} P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\ &= -3At^2 + (4A - 3B)t + 2A + 2B - 3C \end{aligned}$$

- Compare the coefficients one has

$$\begin{aligned} & -3A = 1, 4A - 3B = 0, 2A + 2B - 3C = 0 \\ \Rightarrow \quad & A = -1/3, B = -4/9, C = (2A + 2B)/3 = -14/27. \\ \Rightarrow \quad & P(t) = -\frac{1}{3}t^2 - \frac{4}{9}t - \frac{14}{27}. \end{aligned}$$

- CHECK YOUR SOLUTION (skip).

The ridiculously fabricated example problem

- And finally you combine all the 4 particular solutions

The ridiculously fabricated example problem

- And finally you combine all the 4 particular solutions together with the complementary solution,

The ridiculously fabricated example problem

- And finally you combine all the 4 particular solutions together with the complementary solution, to get the general solution

$$y(t) = C_1 e^{t} + C_2 e^{-3t}$$

The ridiculously fabricated example problem

- And finally you combine all the 4 particular solutions together with the complementary solution, to get the general solution

$$y(t) = C_1 e^t + C_2 e^{-3t} + (\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)e^t$$

The ridiculously fabricated example problem

- And finally you combine all the 4 particular solutions together with the complementary solution, to get the general solution

$$\begin{aligned} y(t) &= C_1 e^t + C_2 e^{-3t} + (\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)e^t \\ &+ e^{-3t}(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t) \end{aligned}$$

The ridiculously fabricated example problem

- And finally you combine all the 4 particular solutions together with the complementary solution, to get the general solution

$$\begin{aligned} y(t) &= C_1 e^t + C_2 e^{-3t} + (\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)e^t \\ &+ e^{-3t}(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t) - \frac{19}{425}\cos 4t + \frac{8}{425}\sin 4t \end{aligned}$$

The ridiculously fabricated example problem

- And finally you combine all the 4 particular solutions together with the complementary solution, to get the general solution

$$
\begin{aligned}
y(t) &= C_1 e^t + C_2 e^{-3t} + (\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t)e^t \\
&+ e^{-3t}(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t) - \frac{19}{425}\cos 4t + \frac{8}{425}\sin 4t \\
&- \frac{1}{3}t^2 - \frac{4}{9}t - \frac{14}{27}.
\end{aligned}
$$

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation,

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

  if you are as lazy as me,

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

  if you are as lazy as me, you can just try $P(t) = t$

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

if you are as lazy as me, you can just try $P(t) = t$ and bingo.

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

  if you are as lazy as me, you can just try $P(t) = t$ and bingo. Or you can proceed by trying $P(t) = Ct + D$

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

  if you are as lazy as me, you can just try $P(t) = t$ and bingo. Or you can proceed by trying $P(t) = Ct + D$ to get $C = 1, D = 0$.

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

  if you are as lazy as me, you can just try $P(t) = t$ and bingo. Or you can proceed by trying $P(t) = Ct + D$ to get $C = 1, D = 0$.
- Now we deal the first part,

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

  if you are as lazy as me, you can just try $P(t) = t$ and bingo. Or you can proceed by trying $P(t) = Ct + D$ to get $C = 1, D = 0$.
- Now we deal the first part, namely

$$y'' + y = \cos t.$$

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

  if you are as lazy as me, you can just try $P(t) = t$ and bingo. Or you can proceed by trying $P(t) = Ct + D$ to get $C = 1, D = 0$.
- Now we deal the first part, namely

$$y'' + y = \cos t.$$

- The template for the first try

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

  if you are as lazy as me, you can just try $P(t) = t$ and bingo. Or you can proceed by trying $P(t) = Ct + D$ to get $C = 1, D = 0$.
- Now we deal the first part, namely

$$y'' + y = \cos t.$$

- The template for the first try is $P(t) = A \cos t + B \sin t$.

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

  if you are as lazy as me, you can just try $P(t) = t$ and bingo. Or you can proceed by trying $P(t) = Ct + D$ to get $C = 1, D = 0$.
- Now we deal the first part, namely

$$y'' + y = \cos t.$$

- The template for the first try is $P(t) = A\cos t + B\sin t$. But this is part of the complementary solution.

Quiz Problem 2

Find the general solution to the differential equation

$$y'' + y = \cos t + t$$

- The complementary solution is

$$y(t) = c_1 \cos t + c_2 \sin t.$$

- For the second part of the equation, namely

$$y'' + y = t,$$

  if you are as lazy as me, you can just try $P(t) = t$ and bingo. Or you can proceed by trying $P(t) = Ct + D$ to get $C = 1, D = 0$.
- Now we deal the first part, namely

$$y'' + y = \cos t.$$

- The template for the first try is $P(t) = A\cos t + B\sin t$. But this is part of the complementary solution. Therefore it is immediate that the first try fails.

Quiz Problem 2

- Multiply your template by another t.

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class.

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t,$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$P(t) \quad = \quad At\cos t + Bt\sin t,$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$\begin{aligned} P(t) &= At\cos t + Bt\sin t, \\ P'(t) &= A(\cos t - t\sin t) \end{aligned}$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$\begin{aligned} P(t) &= At\cos t + Bt\sin t, \\ P'(t) &= A(\cos t - t\sin t) + B(\sin t + t\cos t) \end{aligned}$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$\begin{aligned} P(t) &= At\cos t + Bt\sin t, \\ P'(t) &= A(\cos t - t\sin t) + B(\sin t + t\cos t) \\ &= A\cos t + B\sin t + Bt\cos t - At\sin t \end{aligned}$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$\begin{aligned} P(t) &= At\cos t + Bt\sin t, \\ P'(t) &= A(\cos t - t\sin t) + B(\sin t + t\cos t) \\ &= A\cos t + B\sin t + Bt\cos t - At\sin t \\ P''(t) &= -A\sin t \end{aligned}$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$\begin{aligned} P(t) &= At\cos t + Bt\sin t, \\ P'(t) &= A(\cos t - t\sin t) + B(\sin t + t\cos t) \\ &= A\cos t + B\sin t + Bt\cos t - At\sin t \\ P''(t) &= -A\sin t + B\cos t \end{aligned}$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$\begin{aligned}
P(t) &= At\cos t + Bt\sin t, \\
P'(t) &= A(\cos t - t\sin t) + B(\sin t + t\cos t) \\
&= A\cos t + B\sin t + Bt\cos t - At\sin t \\
P''(t) &= -A\sin t + B\cos t + B(\cos t - t\sin t)
\end{aligned}$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$\begin{aligned} P(t) &= At\cos t + Bt\sin t, \\ P'(t) &= A(\cos t - t\sin t) + B(\sin t + t\cos t) \\ &= A\cos t + B\sin t + Bt\cos t - At\sin t \\ P''(t) &= -A\sin t + B\cos t + B(\cos t - t\sin t) - A(\sin t + t\cos t) \end{aligned}$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$\begin{aligned}
P(t) &= At\cos t + Bt\sin t, \\
P'(t) &= A(\cos t - t\sin t) + B(\sin t + t\cos t) \\
&= A\cos t + B\sin t + Bt\cos t - At\sin t \\
P''(t) &= -A\sin t + B\cos t + B(\cos t - t\sin t) - A(\sin t + t\cos t) \\
&= -2A\sin t + 2B\cos t - At\cos t - Bt\sin t.
\end{aligned}$$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$\begin{aligned}
P(t) &= At\cos t + Bt\sin t, \\
P'(t) &= A(\cos t - t\sin t) + B(\sin t + t\cos t) \\
&= A\cos t + B\sin t + Bt\cos t - At\sin t \\
P''(t) &= -A\sin t + B\cos t + B(\cos t - t\sin t) - A(\sin t + t\cos t) \\
&= -2A\sin t + 2B\cos t - At\cos t - Bt\sin t.
\end{aligned}$$

$P'' + P$

Quiz Problem 2

- Multiply your template by another t. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t\cos t)' = \cos t - t\sin t, (t\sin t)' = \sin t + t\cos t$$

- Now let's get P, P', P'':

$$\begin{aligned}
P(t) &= At\cos t + Bt\sin t, \\
P'(t) &= A(\cos t - t\sin t) + B(\sin t + t\cos t) \\
&= A\cos t + B\sin t + Bt\cos t - At\sin t \\
P''(t) &= -A\sin t + B\cos t + B(\cos t - t\sin t) - A(\sin t + t\cos t) \\
&= -2A\sin t + 2B\cos t - At\cos t - Bt\sin t. \\
P'' + P &= -2A\sin t + 2B\cos t
\end{aligned}$$

Quiz Problem 2

- Compare the coefficients

Quiz Problem 2

- Compare the coefficients

$$-2A\sin t + 2B\cos t = \cos t$$

Quiz Problem 2

- Compare the coefficients

$$\begin{aligned} & -2A\sin t + 2B\cos t = \cos t \\ \Rightarrow \quad & -2A = 0, 2B = 1 \end{aligned}$$

Quiz Problem 2

- Compare the coefficients

$$
\begin{aligned}
& -2A\sin t + 2B\cos t = \cos t \\
\Rightarrow\ & -2A = 0, 2B = 1 \\
\Rightarrow\ & A = 0, B = 1/2.
\end{aligned}
$$

Quiz Problem 2

- Compare the coefficients

$$
\begin{aligned}
& -2A\sin t + 2B\cos t = \cos t \\
\Rightarrow\quad & -2A = 0, 2B = 1 \\
\Rightarrow\quad & A = 0, B = 1/2.
\end{aligned}
$$

- So the particular solution we are looking for

Quiz Problem 2

- Compare the coefficients

$$\begin{aligned} & -2A\sin t + 2B\cos t = \cos t \\ \Rightarrow \quad & -2A = 0, 2B = 1 \\ \Rightarrow \quad & A = 0, B = 1/2. \end{aligned}$$

- So the particular solution we are looking for is

$$P(t) = \frac{1}{2} t \sin t$$

Quiz Problem 2

- Compare the coefficients

$$\begin{aligned} & -2A\sin t + 2B\cos t = \cos t \\ \Rightarrow \quad & -2A = 0, 2B = 1 \\ \Rightarrow \quad & A = 0, B = 1/2. \end{aligned}$$

- So the particular solution we are looking for is

$$P(t) = \frac{1}{2} t \sin t$$

- So the general solution for the whole ODE

Quiz Problem 2

- Compare the coefficients

$$\begin{aligned} & -2A\sin t + 2B\cos t = \cos t \\ \Rightarrow \quad & -2A = 0, 2B = 1 \\ \Rightarrow \quad & A = 0, B = 1/2. \end{aligned}$$

- So the particular solution we are looking for is

$$P(t) = \frac{1}{2} t \sin t$$

- So the general solution for the whole ODE is

$$y(t) = C_1 \cos t + C_2 \sin t + \frac{1}{2} t \sin t + t$$

Quiz Problem 2

- Compare the coefficients

$$\begin{aligned} & -2A\sin t + 2B\cos t = \cos t \\ \Rightarrow\ & -2A = 0, 2B = 1 \\ \Rightarrow\ & A = 0, B = 1/2. \end{aligned}$$

- So the particular solution we are looking for is

$$P(t) = \frac{1}{2} t \sin t$$

- So the general solution for the whole ODE is

$$y(t) = C_1 \cos t + C_2 \sin t + \frac{1}{2} t \sin t + t$$

- Remember to check your solution.

Quiz Problem 2: Remarks

- It is not impossible to generalize the exponential-shift rule to simplify the computation.

Quiz Problem 2: Remarks

- It is not impossible to generalize the exponential-shift rule to simplify the computation. However, the generalization requires

Quiz Problem 2: Remarks

- It is not impossible to generalize the exponential-shift rule to simplify the computation. However, the generalization requires that you have a substantial understanding of complex functions

Quiz Problem 2: Remarks

- It is not impossible to generalize the exponential-shift rule to simplify the computation. However, the generalization requires that you have a substantial understanding of complex functions and the process of complexification.

Quiz Problem 2: Remarks

- It is not impossible to generalize the exponential-shift rule to simplify the computation. However, the generalization requires that you have a substantial understanding of complex functions and the process of complexification. Considering your workload,

Quiz Problem 2: Remarks

- It is not impossible to generalize the exponential-shift rule to simplify the computation. However, the generalization requires that you have a substantial understanding of complex functions and the process of complexification. Considering your workload, I decide not to introduce it here.

Quiz Problem 2: Remarks

- It is not impossible to generalize the exponential-shift rule to simplify the computation. However, the generalization requires that you have a substantial understanding of complex functions and the process of complexification. Considering your workload, I decide not to introduce it here. But anyone who is very interested shall contact me.

Quiz Problem 2: Remarks

- It is not impossible to generalize the exponential-shift rule to simplify the computation. However, the generalization requires that you have a substantial understanding of complex functions and the process of complexification. Considering your workload, I decide not to introduce it here. But anyone who is very interested shall contact me. I'll either teach in person or prepare some additional slides.

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture,

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t)$$

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)}$$

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

and the particular solution is simply the real part,

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

and the particular solution is simply the real part, namely

$$P(t) = \frac{1}{2}t\sin t$$

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

  and the particular solution is simply the real part, namely

$$P(t) = \frac{1}{2} t \sin t$$

- You can actually see the technique issue of complexification:

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

and the particular solution is simply the real part, namely

$$P(t) = \frac{1}{2} t \sin t$$

- You can actually see the technique issue of complexification: you have to be very clear

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

and the particular solution is simply the real part, namely

$$P(t) = \frac{1}{2}t\sin t$$

- You can actually see the technique issue of complexification: you have to be very clear that when you shall take real part

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

and the particular solution is simply the real part, namely

$$P(t) = \frac{1}{2}t\sin t$$

- You can actually see the technique issue of complexification: you have to be very clear that when you shall take real part and when you shall take complex part.

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

and the particular solution is simply the real part, namely

$$P(t) = \frac{1}{2}t\sin t$$

- You can actually see the technique issue of complexification: you have to be very clear that when you shall take real part and when you shall take complex part. The slightest confusion or mistake will give a wrong solution.

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

and the particular solution is simply the real part, namely

$$P(t) = \frac{1}{2}t\sin t$$

- You can actually see the technique issue of complexification: you have to be very clear that when you shall take real part and when you shall take complex part. The slightest confusion or mistake will give a wrong solution. That's why I don't want to talk about it here.

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

and the particular solution is simply the real part, namely

$$P(t) = \frac{1}{2}t\sin t$$

- You can actually see the technique issue of complexification: you have to be very clear that when you shall take real part and when you shall take complex part. The slightest confusion or mistake will give a wrong solution. That's why I don't want to talk about it here. After several weeks of heavy courseload elsewhere,

Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck's lecture, then you immediate get

$$\tilde{P}(t) = \frac{te^{it}}{f'(i)} = \frac{t\cos t + it\sin t}{2i}$$

and the particular solution is simply the real part, namely

$$P(t) = \frac{1}{2}t\sin t$$

- You can actually see the technique issue of complexification: you have to be very clear that when you shall take real part and when you shall take complex part. The slightest confusion or mistake will give a wrong solution. That's why I don't want to talk about it here. After several weeks of heavy courseload elsewhere, you may forget the right way of doing it.

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it.

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it. The second term would be easy:

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it. The second term would be easy: Just put in $P(t) = A$

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it. The second term would be easy: Just put in $P(t) = A$ and by $P'' + 9P$

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it. The second term would be easy: Just put in $P(t) = A$ and by $P'' + 9P = 9A$

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it. The second term would be easy: Just put in $P(t) = A$ and by $P'' + 9P = 9A = 6$

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it. The second term would be easy: Just put in $P(t) = A$ and by $P'' + 9P = 9A = 6$ one has $P(t) = 2/3$.

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it. The second term would be easy: Just put in $P(t) = A$ and by $P'' + 9P = 9A = 6$ one has $P(t) = 2/3$.
- Let's look at the first term.

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it. The second term would be easy: Just put in $P(t) = A$ and by $P'' + 9P = 9A = 6$ one has $P(t) = 2/3$.
- Let's look at the first term. The template for the first try

Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it. The second term would be easy: Just put in $P(t) = A$ and by $P'' + 9P = 9A = 6$ one has $P(t) = 2/3$.
- Let's look at the first term. The template for the first try is

$$P(t) = e^{3t}(At^2 + Bt + C)$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$P'' + 9P$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$P'' + 9P = (D^2 + 9)$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2+9)e^{3t}(At^2+Bt+C) \\ &= e^{3t}((D+3)^2+9)(At^2+Bt+C) \end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2 + 9)e^{3t}(At^2 + Bt + C) \\ &= e^{3t}((D+3)^2 + 9)(At^2 + Bt + C) \\ &= e^{3t} \end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2 + 9)e^{3t}(At^2 + Bt + C) \\ &= e^{3t}((D+3)^2 + 9)(At^2 + Bt + C) \\ &= e^{3t}(D^2 + 6D + 18) \end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2 + 9)e^{3t}(At^2 + Bt + C) \\ &= e^{3t}((D+3)^2 + 9)(At^2 + Bt + C) \\ &= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2 + 9)e^{3t}(At^2 + Bt + C) \\ &= e^{3t}((D+3)^2 + 9)(At^2 + Bt + C) \\ &= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \\ &= e^{3t} \end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2 + 9)e^{3t}(At^2 + Bt + C) \\ &= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C) \\ &= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \\ &= e^{3t}(2A \end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2+9)e^{3t}(At^2+Bt+C) \\ &= e^{3t}((D+3)^2+9)(At^2+Bt+C) \\ &= e^{3t}(D^2+6D+18)(At^2+Bt+C) \\ &= e^{3t}(2A+6(2At+B) \end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$
\begin{aligned}
P'' + 9P &= (D^2 + 9)e^{3t}(At^2 + Bt + C) \\
&= e^{3t}((D+3)^2 + 9)(At^2 + Bt + C) \\
&= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \\
&= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C))
\end{aligned}
$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned}
P'' + 9P &= (D^2 + 9)e^{3t}(At^2 + Bt + C) \\
&= e^{3t}((D+3)^2 + 9)(At^2 + Bt + C) \\
&= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \\
&= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C)) \\
&= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C
\end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2+9)e^{3t}(At^2+Bt+C) \\ &= e^{3t}((D+3)^2+9)(At^2+Bt+C) \\ &= e^{3t}(D^2+6D+18)(At^2+Bt+C) \\ &= e^{3t}(2A+6(2At+B)+18(At^2+Bt+C)) \\ &= e^{3t}(18At^2+(12A+18B)t+2A+6B+18C \end{aligned}$$

- Compare coefficients

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2+9)e^{3t}(At^2+Bt+C) \\ &= e^{3t}((D+3)^2+9)(At^2+Bt+C) \\ &= e^{3t}(D^2+6D+18)(At^2+Bt+C) \\ &= e^{3t}(2A+6(2At+B)+18(At^2+Bt+C)) \\ &= e^{3t}(18At^2+(12A+18B)t+2A+6B+18C \end{aligned}$$

- Compare coefficients

$$e^{3t}(18At^2+(12A+18B)t+2A+6B+18C$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2+9)e^{3t}(At^2+Bt+C) \\ &= e^{3t}((D+3)^2+9)(At^2+Bt+C) \\ &= e^{3t}(D^2+6D+18)(At^2+Bt+C) \\ &= e^{3t}(2A+6(2At+B)+18(At^2+Bt+C)) \\ &= e^{3t}(18At^2+(12A+18B)t+2A+6B+18C \end{aligned}$$

- Compare coefficients

$$e^{3t}(18At^2+(12A+18B)t+2A+6B+18C = t^2e^{3t}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned}
P'' + 9P &= (D^2 + 9)e^{3t}(At^2 + Bt + C) \\
&= e^{3t}((D+3)^2 + 9)(At^2 + Bt + C) \\
&= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \\
&= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C)) \\
&= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C
\end{aligned}$$

- Compare coefficients

$$\begin{aligned}
& e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C = t^2 e^{3t} \\
\Rightarrow \quad & 18A = 1,
\end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2+9)e^{3t}(At^2+Bt+C) \\ &= e^{3t}((D+3)^2+9)(At^2+Bt+C) \\ &= e^{3t}(D^2+6D+18)(At^2+Bt+C) \\ &= e^{3t}(2A+6(2At+B)+18(At^2+Bt+C)) \\ &= e^{3t}(18At^2+(12A+18B)t+2A+6B+18C \end{aligned}$$

- Compare coefficients

$$\begin{aligned} & e^{3t}(18At^2+(12A+18B)t+2A+6B+18C = t^2e^{3t} \\ \Rightarrow\quad & 18A = 1, 12A+18B = 0, \end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2+9)e^{3t}(At^2+Bt+C) \\ &= e^{3t}((D+3)^2+9)(At^2+Bt+C) \\ &= e^{3t}(D^2+6D+18)(At^2+Bt+C) \\ &= e^{3t}(2A+6(2At+B)+18(At^2+Bt+C)) \\ &= e^{3t}(18At^2+(12A+18B)t+2A+6B+18C \end{aligned}$$

- Compare coefficients

$$\begin{aligned} & e^{3t}(18At^2+(12A+18B)t+2A+6B+18C = t^2e^{3t} \\ \Rightarrow \quad & 18A = 1, 12A+18B = 0, 2A+6B+18C = 0 \end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned}
P'' + 9P &= (D^2 + 9)e^{3t}(At^2 + Bt + C) \\
&= e^{3t}((D+3)^2 + 9)(At^2 + Bt + C) \\
&= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \\
&= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C)) \\
&= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C
\end{aligned}$$

- Compare coefficients

$$\begin{aligned}
& e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C = t^2 e^{3t} \\
\Rightarrow \quad & 18A = 1, 12A + 18B = 0, 2A + 6B + 18C = 0 \\
\Rightarrow \quad & A = 1/18,
\end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2+9)e^{3t}(At^2+Bt+C) \\ &= e^{3t}((D+3)^2+9)(At^2+Bt+C) \\ &= e^{3t}(D^2+6D+18)(At^2+Bt+C) \\ &= e^{3t}(2A+6(2At+B)+18(At^2+Bt+C)) \\ &= e^{3t}(18At^2+(12A+18B)t+2A+6B+18C \end{aligned}$$

- Compare coefficients

$$\begin{aligned} & e^{3t}(18At^2+(12A+18B)t+2A+6B+18C = t^2e^{3t} \\ \Rightarrow\ & 18A=1, 12A+18B=0, 2A+6B+18C=0 \\ \Rightarrow\ & A=1/18, B=-2A/3=-1/27, \end{aligned}$$

Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$\begin{aligned} P'' + 9P &= (D^2+9)e^{3t}(At^2+Bt+C) \\ &= e^{3t}((D+3)^2+9)(At^2+Bt+C) \\ &= e^{3t}(D^2+6D+18)(At^2+Bt+C) \\ &= e^{3t}(2A+6(2At+B)+18(At^2+Bt+C)) \\ &= e^{3t}(18At^2+(12A+18B)t+2A+6B+18C \end{aligned}$$

- Compare coefficients

$$\begin{aligned} & e^{3t}(18At^2+(12A+18B)t+2A+6B+18C = t^2e^{3t} \\ \Rightarrow\ & 18A = 1, 12A+18B = 0, 2A+6B+18C = 0 \\ \Rightarrow\ & A = 1/18, B = -2A/3 = -1/27, C = -(2A+6B)/18 = 1/162 \end{aligned}$$

Homework Problem 3.5.7

- So the particular solution we are looking for

Homework Problem 3.5.7

- So the particular solution we are looking for is

$$P(t) = e^{3t}(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162})$$

- So the particular solution we are looking for is

$$P(t) = e^{3t}(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162})$$

- CHECK! (skipped)

Homework Problem 3.5.7

- So the particular solution we are looking for is

$$P(t) = e^{3t}(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162})$$

- CHECK! (skipped)
- Combined with the results above,

Homework Problem 3.5.7

- So the particular solution we are looking for is

$$P(t) = e^{3t}(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162})$$

- CHECK! (skipped)
- Combined with the results above, the general solution

Homework Problem 3.5.7

- So the particular solution we are looking for is

$$P(t) = e^{3t}(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162})$$

- CHECK! (skipped)
- Combined with the results above, the general solution is

$$y(t) = C_1 \cos 3t + C_2 \sin 3t$$

Homework Problem 3.5.7

- So the particular solution we are looking for is

$$P(t) = e^{3t}(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162})$$

- CHECK! (skipped)
- Combined with the results above, the general solution is

$$\begin{aligned} y(t) &= C_1 \cos 3t + C_2 \sin 3t \\ &+ e^{3t}(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162}) \end{aligned}$$

Homework Problem 3.5.7

- So the particular solution we are looking for is

$$P(t) = e^{3t}\left(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162}\right)$$

- CHECK! (skipped)
- Combined with the results above, the general solution is

$$\begin{aligned} y(t) &= C_1 \cos 3t + C_2 \sin 3t \\ &+ e^{3t}\left(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162}\right) + \frac{2}{3} \end{aligned}$$

Homework Problem 3.5.12

Find the general solution of

$$y'' + \omega_0^2 y = \cos \omega t$$

Homework Problem 3.5.12

Find the general solution of

$$y'' + \omega_0^2 y = \cos \omega t$$

- This equation is the resonance equation discussed in great detail in MIT Lecture 14.

Homework Problem 3.5.12

Find the general solution of

$$y'' + \omega_0^2 y = \cos \omega t$$

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it.

Homework Problem 3.5.12

Find the general solution of

$$y'' + \omega_0^2 y = \cos \omega t$$

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it. I am here just recording my boardwork when teaching Section 10.

Homework Problem 3.5.12

Find the general solution of

$$y'' + \omega_0^2 y = \cos \omega t$$

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it. I am here just recording my boardwork when teaching Section 10.
- The complementary solution to this ODE

Homework Problem 3.5.12

Find the general solution of

$$y'' + \omega_0^2 y = \cos \omega t$$

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it. I am here just recording my boardwork when teaching Section 10.
- The complementary solution to this ODE is

$$C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$$

Homework Problem 3.5.12

Find the general solution of

$$y'' + \omega_0^2 y = \cos \omega t$$

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it. I am here just recording my boardwork when teaching Section 10.
- The complementary solution to this ODE is

$$C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$$

- The template for the first try

Homework Problem 3.5.12

Find the general solution of

$$y'' + \omega_0^2 y = \cos \omega t$$

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it. I am here just recording my boardwork when teaching Section 10.
- The complementary solution to this ODE is

$$C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$$

- The template for the first try is

$$P(t) = A \cos \omega_0 t + B \sin \omega_0 t.$$

Homework Problem 3.5.12

Find the general solution of

$$y'' + \omega_0^2 y = \cos \omega t$$

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it. I am here just recording my boardwork when teaching Section 10.
- The complementary solution to this ODE is

$$C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$$

- The template for the first try is

$$P(t) = A \cos \omega_0 t + B \sin \omega_0 t.$$

But this is part of the complementary solution,

Homework Problem 3.5.12

Find the general solution of

$$y'' + \omega_0^2 y = \cos \omega t$$

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it. I am here just recording my boardwork when teaching Section 10.
- The complementary solution to this ODE is

$$C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$$

- The template for the first try is

$$P(t) = A \cos \omega_0 t + B \sin \omega_0 t.$$

  But this is part of the complementary solution, therefore the first try fails.

Homework Problem 3.5.12

- Multiply by t

Homework Problem 3.5.12

- Multiply by t and try

$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$

Homework Problem 3.5.12

- Multiply by t and try

$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$

For convenience of use below,

Homework Problem 3.5.12

- Multiply by t and try

$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$

For convenience of use below, note that

$$(t\cos\omega_0 t)' = \cos\omega_0 t - \omega_0 t\sin\omega_0 t$$

Homework Problem 3.5.12

- Multiply by t and try

$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$

For convenience of use below, note that

$$\begin{aligned}(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t \\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t\end{aligned}$$

Homework Problem 3.5.12

- Multiply by t and try
$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$
For convenience of use below, note that
$$\begin{aligned}(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t \\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t\end{aligned}$$
- Now get all the derivatives:

Homework Problem 3.5.12

- Multiply by t and try
$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$
For convenience of use below, note that
$$\begin{aligned}(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t\\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t\end{aligned}$$
- Now get all the derivatives:
$$P'(t) = A(\cos\omega_0 t - \omega_0 t\sin\omega_0 t)$$

Homework Problem 3.5.12

- Multiply by t and try
$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$
For convenience of use below, note that
$$\begin{aligned}(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t\\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t\end{aligned}$$
- Now get all the derivatives:
$$P'(t) = A(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) + B(\sin\omega_0 t + \omega_0 t\cos\omega_0 t)$$

Homework Problem 3.5.12

- Multiply by t and try
$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$
For convenience of use below, note that
$$\begin{aligned}(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t\\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t\end{aligned}$$
- Now get all the derivatives:
$$\begin{aligned}P'(t) &= A(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) + B(\sin\omega_0 t + \omega_0 t\cos\omega_0 t)\\ &= A\cos\omega_0 t + B\sin\omega_0 t + B\omega_0 t\cos\omega_0 t - A\omega_0 t\sin\omega_0 t\end{aligned}$$

Homework Problem 3.5.12

- Multiply by t and try
$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$
For convenience of use below, note that
$$\begin{aligned}(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t\\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t\end{aligned}$$
- Now get all the derivatives:
$$\begin{aligned}P'(t) &= A(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) + B(\sin\omega_0 t + \omega_0 t\cos\omega_0 t)\\ &= A\cos\omega_0 t + B\sin\omega_0 t + B\omega_0 t\cos\omega_0 t - A\omega_0 t\sin\omega_0 t\\ P''(t) &= -A\omega_0\sin\omega_0 t\end{aligned}$$

Homework Problem 3.5.12

- Multiply by t and try

$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$

For convenience of use below, note that

$$\begin{aligned}(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t\\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t\end{aligned}$$

- Now get all the derivatives:

$$\begin{aligned}P'(t) &= A(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) + B(\sin\omega_0 t + \omega_0 t\cos\omega_0 t)\\ &= A\cos\omega_0 t + B\sin\omega_0 t + B\omega_0 t\cos\omega_0 t - A\omega_0 t\sin\omega_0 t\\ P''(t) &= -A\omega_0\sin\omega_0 t + B\omega\cos\omega_0 t\end{aligned}$$

Homework Problem 3.5.12

- Multiply by t and try

$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$

For convenience of use below, note that

$$\begin{aligned}(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t\\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t\end{aligned}$$

- Now get all the derivatives:

$$\begin{aligned}P'(t) &= A(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) + B(\sin\omega_0 t + \omega_0 t\cos\omega_0 t)\\ &= A\cos\omega_0 t + B\sin\omega_0 t + B\omega_0 t\cos\omega_0 t - A\omega_0 t\sin\omega_0 t\\ P''(t) &= -A\omega_0\sin\omega_0 t + B\omega\cos\omega_0 t\\ &+ B\omega_0(\cos\omega_0 t - \omega_0 t\sin\omega_0 t)\end{aligned}$$

Homework Problem 3.5.12

- Multiply by t and try
$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$
For convenience of use below, note that
$$\begin{aligned}(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t \\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t\end{aligned}$$
- Now get all the derivatives:

$$\begin{aligned}P'(t) &= A(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) + B(\sin\omega_0 t + \omega_0 t\cos\omega_0 t) \\ &= A\cos\omega_0 t + B\sin\omega_0 t + B\omega_0 t\cos\omega_0 t - A\omega_0 t\sin\omega_0 t \\ P''(t) &= -A\omega_0\sin\omega_0 t + B\omega\cos\omega_0 t \\ &+ B\omega_0(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) - A\omega(\sin\omega_0 t + \omega_0 t\cos\omega_0 t)\end{aligned}$$

Homework Problem 3.5.12

- Multiply by t and try

$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$

For convenience of use below, note that

$$\begin{aligned}
(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t \\
(t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t
\end{aligned}$$

- Now get all the derivatives:

$$\begin{aligned}
P'(t) &= A(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) + B(\sin\omega_0 t + \omega_0 t\cos\omega_0 t) \\
&= A\cos\omega_0 t + B\sin\omega_0 t + B\omega_0 t\cos\omega_0 t - A\omega_0 t\sin\omega_0 t \\
P''(t) &= -A\omega_0\sin\omega_0 t + B\omega\cos\omega_0 t \\
&+ B\omega_0(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) - A\omega(\sin\omega_0 t + \omega_0 t\cos\omega_0 t) \\
&= -2A\omega_0\sin\omega_0 t + 2B\omega_0\cos\omega_0 t - A\omega_0^2\cos\omega_0 t - B\omega_0^2 t\sin\omega_0 t
\end{aligned}$$

Homework Problem 3.5.12

- Multiply by t and try

$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$

For convenience of use below, note that

$$\begin{aligned} (t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t \\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t \end{aligned}$$

- Now get all the derivatives:

$$\begin{aligned} P'(t) &= A(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) + B(\sin\omega_0 t + \omega_0 t\cos\omega_0 t) \\ &= A\cos\omega_0 t + B\sin\omega_0 t + B\omega_0 t\cos\omega_0 t - A\omega_0 t\sin\omega_0 t \\ P''(t) &= -A\omega_0\sin\omega_0 t + B\omega\cos\omega_0 t \\ &+ B\omega_0(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) - A\omega(\sin\omega_0 t + \omega_0 t\cos\omega_0 t) \\ &= -2A\omega_0\sin\omega_0 t + 2B\omega_0\cos\omega_0 t - A\omega_0^2\cos\omega_0 t - B\omega_0^2 t\sin\omega_0 t \end{aligned}$$

So $P'' + \omega_0^2 P$

Homework Problem 3.5.12

- Multiply by t and try
$$P(t) = At\cos\omega_0 t + Bt\sin\omega_0 t.$$
For convenience of use below, note that
$$\begin{aligned}(t\cos\omega_0 t)' &= \cos\omega_0 t - \omega_0 t\sin\omega_0 t\\ (t\sin\omega_0 t)' &= \sin\omega_0 t + \omega_0 t\cos\omega_0 t\end{aligned}$$
- Now get all the derivatives:
$$\begin{aligned}P'(t) &= A(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) + B(\sin\omega_0 t + \omega_0 t\cos\omega_0 t)\\ &= A\cos\omega_0 t + B\sin\omega_0 t + B\omega_0 t\cos\omega_0 t - A\omega_0 t\sin\omega_0 t\\ P''(t) &= -A\omega_0\sin\omega_0 t + B\omega\cos\omega_0 t\\ &+ B\omega_0(\cos\omega_0 t - \omega_0 t\sin\omega_0 t) - A\omega(\sin\omega_0 t + \omega_0 t\cos\omega_0 t)\\ &= -2A\omega_0\sin\omega_0 t + 2B\omega_0\cos\omega_0 t - A\omega_0^2\cos\omega_0 t - B\omega_0^2 t\sin\omega_0 t\end{aligned}$$
So $P'' + \omega_0^2 P = -2A\omega_0\sin\omega_0 t + 2B\omega_0\cos\omega_0 t.$

Homework Problem 3.5.12

- Compare the coefficients:

Homework Problem 3.5.12

- Compare the coefficients:

$$-2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t$$

Homework Problem 3.5.12

- Compare the coefficients:

$$-2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t = \cos \omega_0 t$$

Homework Problem 3.5.12

- Compare the coefficients:

$$
\begin{aligned}
& -2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t = \cos \omega_0 t \\
\Rightarrow \quad & A = 0, B = 1/(2\omega_0).
\end{aligned}
$$

Homework Problem 3.5.12

- Compare the coefficients:

$$-2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t = \cos \omega_0 t$$
$$\Rightarrow \quad A = 0, B = 1/(2\omega_0).$$

So

$$P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t$$

.

Homework Problem 3.5.12

- Compare the coefficients:

$$
\begin{aligned}
& -2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t = \cos \omega_0 t \\
\Rightarrow \quad & A = 0, B = 1/(2\omega_0).
\end{aligned}
$$

  So

$$P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t$$

  .

- So the general solution of this ODE is

$$C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$$

Homework Problem 3.5.12

- Compare the coefficients:

$$
\begin{aligned}
& -2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t = \cos \omega_0 t \\
\Rightarrow \quad & A = 0, B = 1/(2\omega_0).
\end{aligned}
$$

  So

$$P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t$$

  .

- So the general solution of this ODE is

$$C_1 \cos \omega_0 t + C_2 \sin \omega_0 t + \frac{1}{2\omega_0} t \sin \omega_0 t.$$

Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem,

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t)$$

Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i}$$

Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t\cos\omega_0 t + it\sin\omega_0 t}{2\omega_0 i}$$

Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t\cos\omega_0 t + it\sin\omega_0 t}{2\omega_0 i}$$

and the particular solution

Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t\cos\omega_0 t + it\sin\omega_0 t}{2\omega_0 i}$$

  and the particular solution is the real part

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t\cos\omega_0 t + it\sin\omega_0 t}{2\omega_0 i}$$

and the particular solution is the real part

$$P(t) = \frac{1}{2\omega_0} t\sin\omega_0 t$$

Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t\cos\omega_0 t + it\sin\omega_0 t}{2\omega_0 i}$$

  and the particular solution is the real part

$$P(t) = \frac{1}{2\omega_0} t\sin\omega_0 t$$

- When $\omega_0 = 1$,

Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t\cos\omega_0 t + it\sin\omega_0 t}{2\omega_0 i}$$

and the particular solution is the real part

$$P(t) = \frac{1}{2\omega_0} t\sin\omega_0 t$$

- When $\omega_0 = 1$, you should easily recover the first part of the Quiz Problem.

Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t\cos\omega_0 t + it\sin\omega_0 t}{2\omega_0 i}$$

  and the particular solution is the real part

$$P(t) = \frac{1}{2\omega_0} t\sin\omega_0 t$$

- When $\omega_0 = 1$, you should easily recover the first part of the Quiz Problem.
- If the right hand side becomes $\sin\omega_0 t$,

Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t\cos\omega_0 t + it\sin\omega_0 t}{2\omega_0 i}$$

  and the particular solution is the real part

$$P(t) = \frac{1}{2\omega_0} t\sin\omega_0 t$$

- When $\omega_0 = 1$, you should easily recover the first part of the Quiz Problem.
- If the right hand side becomes $\sin\omega_0 t$, you should then take the imaginary part of $\tilde{P}$

Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t\cos\omega_0 t + it\sin\omega_0 t}{2\omega_0 i}$$

  and the particular solution is the real part

$$P(t) = \frac{1}{2\omega_0} t\sin\omega_0 t$$

- When $\omega_0 = 1$, you should easily recover the first part of the Quiz Problem.
- If the right hand side becomes $\sin\omega_0 t$, you should then take the imaginary part of $\tilde{P}$ as your particular solution.

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution is

$$C_1 e^t + C_2 te^t$$

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution is

$$C_1 e^t + C_2 te^t$$

- Again separate it.

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution is

$$C_1 e^t + C_2 te^t$$

- Again separate it. It is immediate

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution is

$$C_1 e^t + C_2 te^t$$

- Again separate it. It is immediate that $P(t) = 4$ is a solution of $y'' - 2y' + y = 4$.

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution is

$$C_1 e^t + C_2 te^t$$

- Again separate it. It is immediate that $P(t) = 4$ is a solution of $y'' - 2y' + y = 4$. So we just focus on the first term.

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution is

$$C_1 e^t + C_2 te^t$$

- Again separate it. It is immediate that $P(t) = 4$ is a solution of $y'' - 2y' + y = 4$. So we just focus on the first term.
- The template for first try

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution is

$$C_1 e^t + C_2 te^t$$

- Again separate it. It is immediate that $P(t) = 4$ is a solution of $y'' - 2y' + y = 4$. So we just focus on the first term.
- The template for first try is $P(t) = e^t(At + B)$.

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution is

$$C_1 e^t + C_2 te^t$$

- Again separate it. It is immediate that $P(t) = 4$ is a solution of $y'' - 2y' + y = 4$. So we just focus on the first term.
- The template for first try is $P(t) = e^t(At + B)$. You should see immediately

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution is

$$C_1 e^t + C_2 te^t$$

- Again separate it. It is immediate that $P(t) = 4$ is a solution of $y'' - 2y' + y = 4$. So we just focus on the first term.
- The template for first try is $P(t) = e^t(At + B)$. You should see immediately that this coincides with the complementary solutions.

Book Problem 3.5.17

Solve the IVP

$$y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 0$$

- The complementary solution is

$$C_1 e^t + C_2 te^t$$

- Again separate it. It is immediate that $P(t) = 4$ is a solution of $y'' - 2y' + y = 4$. So we just focus on the first term.
- The template for first try is $P(t) = e^t(At + B)$. You should see immediately that this coincides with the complementary solutions. so the first try fails.

Book Problem 3.5.17

- Modify your template

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^2 + Bt)$.

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$P'' - 2P' + P$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$P'' - 2P' + P = (D-1)^2$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$P'' - 2P' + P = (D-1)^2(e^t(At^2 + Bt))$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2+Bt)) \\ &= e^t(D+1-1)^2(At^2+Bt) \end{aligned}$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2+Bt)) \\ &= e^t(D+1-1)^2(At^2+Bt) \\ &= e^tD^2(At^2+Bt) \end{aligned}$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2+Bt)) \\ &= e^t(D+1-1)^2(At^2+Bt) \\ &= e^t D^2(At^2+Bt) \\ &= e^t 2A \end{aligned}$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2+Bt)) \\ &= e^t(D+1-1)^2(At^2+Bt) \\ &= e^t D^2(At^2+Bt) \\ &= e^t 2A \end{aligned}$$

There is nothing concerning the te^t.

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2 + Bt)) \\ &= e^t(D+1-1)^2(At^2 + Bt) \\ &= e^t D^2(At^2 + Bt) \\ &= e^t 2A \end{aligned}$$

There is nothing concerning the te^t. So the second try fails.

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$.

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows

$$P'' - 2P' + P$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows

$$P'' - 2P' + P \quad = \quad (D-1)^2(e^t(At^2 + Bt))$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2 + Bt)) \\ &= e^t D^2(At^3 + Bt^2) \end{aligned}$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2 + Bt)) \\ &= e^t D^2(At^3 + Bt^2) \\ &= e^t(6At + 2B) \end{aligned}$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2 + Bt)) \\ &= e^t D^2(At^3 + Bt^2) \\ &= e^t(6At + 2B) \end{aligned}$$

- Compare the coefficients

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2 + Bt)) \\ &= e^t D^2(At^3 + Bt^2) \\ &= e^t(6At + 2B) \end{aligned}$$

- Compare the coefficients

$$e^t(6At + 2B)$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows

$$
\begin{aligned}
P'' - 2P' + P &= (D-1)^2(e^t(At^2 + Bt)) \\
&= e^t D^2(At^3 + Bt^2) \\
&= e^t(6At + 2B)
\end{aligned}
$$

- Compare the coefficients

$$e^t(6At + 2B) = te^t$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2 + Bt)) \\ &= e^t D^2(At^3 + Bt^2) \\ &= e^t(6At + 2B) \end{aligned}$$

- Compare the coefficients

$$\begin{aligned} & e^t(6At + 2B) = te^t \\ \Rightarrow \quad & 6A = 1, 2B = 0 \end{aligned}$$

Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows

$$\begin{aligned} P'' - 2P' + P &= (D-1)^2(e^t(At^2 + Bt)) \\ &= e^t D^2(At^3 + Bt^2) \\ &= e^t(6At + 2B) \end{aligned}$$

- Compare the coefficients

$$\begin{aligned} & e^t(6At + 2B) = te^t \\ \Rightarrow\ & 6A = 1, 2B = 0 \end{aligned}$$

So

$$P(t) = \frac{1}{6}t^3 e^t$$

.

Book Problem 3.5.17

- So the general solution of the ODE

Book Problem 3.5.17

- So the general solution of the ODE is

$$y(t) = C_1 e^t + C_2 t e^t + \frac{1}{6} t^3 e^t + 4$$

Book Problem 3.5.17

- So the general solution of the ODE is

$$y(t) = C_1e^t + C_2te^t + \frac{1}{6}t^3e^t + 4$$

- Put in the initial values,

Book Problem 3.5.17

- So the general solution of the ODE is

$$y(t) = C_1 e^t + C_2 t e^t + \frac{1}{6} t^3 e^t + 4$$

- Put in the initial values, one gets the following equations

Book Problem 3.5.17

- So the general solution of the ODE is

$$y(t) = C_1 e^t + C_2 t e^t + \frac{1}{6} t^3 e^t + 4$$

- Put in the initial values, one gets the following equations

$$C_1 + 4 = 1$$

Book Problem 3.5.17

- So the general solution of the ODE is

$$y(t) = C_1 e^t + C_2 t e^t + \frac{1}{6} t^3 e^t + 4$$

- Put in the initial values, one gets the following equations

$$\begin{aligned} C_1 + 4 &= 1 \\ C_1 + C_2 &= 1 \end{aligned}$$

Book Problem 3.5.17

- So the general solution of the ODE is

$$y(t) = C_1 e^t + C_2 t e^t + \frac{1}{6} t^3 e^t + 4$$

- Put in the initial values, one gets the following equations

$$\begin{aligned} C_1 + 4 &= 1 \\ C_1 + C_2 &= 1 \end{aligned}$$

So $C_1 = -3$, $C_2 = 4$

Book Problem 3.5.17

- So the general solution of the ODE is

$$y(t) = C_1 e^t + C_2 t e^t + \frac{1}{6} t^3 e^t + 4$$

- Put in the initial values, one gets the following equations

$$\begin{aligned} C_1 + 4 &= 1 \\ C_1 + C_2 &= 1 \end{aligned}$$

So $C_1 = -3$, $C_2 = 4$ and thus the solution to the IVP

Book Problem 3.5.17

- So the general solution of the ODE is

$$y(t) = C_1 e^t + C_2 te^t + \frac{1}{6}t^3 e^t + 4$$

- Put in the initial values, one gets the following equations

$$\begin{aligned} C_1 + 4 &= 1 \\ C_1 + C_2 &= 1 \end{aligned}$$

So $C_1 = -3$, $C_2 = 4$ and thus the solution to the IVP is

$$y(t) = -3e^t + 4te^t + \frac{1}{6}t^3 e^t + 4$$

The End